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AN

ELEMENTARY EXPOSITION

OF THE DOCTRINE OF

ENERGY

BY

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PREFACE.

THIS TREATISE is little more than the substance of a set of Lectures given by me in the summer of 1872, at the request of the Head Master, to the 6th Form Boys of the Surrey County School.

My aim was to lay before them—lads of from sixteen to eighteen, soon about to quit school for active life—not a popular account of recent discoveries or theories, such as abound in periodicals and books for the young, but such an exhibition of the principles and methods of modern science, and of some results obtained therefrom, as might show them the use of the mathematical groundwork in which they had been carefully instructed, and perhaps induce some of them to pursue their own scientific education after leaving.

Finding afterwards that the subject was to be made part of the annual examination in 1873, I made up my notes, that the Examiner might see what the course had been. He thought they might be usefully worked up into a fuller treatise: I took advice from other competent judges; and on the whole thought the experiment worth trying.

Since that time various works have appeared in English, going over much of the same ground. But I have not seen any the object of which seemed to be precisely that at which I aimed. I hope, therefore, that, unless the fault be in the execution, my work may still be found useful.

I may add that I have received not only encouragement, but also much valuable advice and criticism, from those whom I have consulted; more especially from my friend the Rev. H. W. Watson, who kindly looked over the whole treatise in its original draft. But, as I have not always accepted the suggestions made to me, but have adhered to such methods of exposition as seemed, after consideration, most satisfactory to my own mind, I am entirely responsible, not only for errors of execution, but for the plan of the whole work.

The Doctrine of Energy—of its 'Conservation,' and the 'Correlation' of its several kinds—is one that binds together all the Physical and Physiological Sciences, by showing that a principle pervades them all which is strictly analogous, perhaps identical, with a long-known Proposition in Dynamics.

It appears to me, therefore, that it cannot be satisfactorily taught, or steadily apprehended, except as an extension or application of that science. And as I do not suppose it reasonable to assume a full, or, indeed, much knowledge of Dynamics in schoolboys, even of the highest Forms, I think it absolutely necessary to begin with going pretty thoroughly into first principles, so as to make the conceptions of 'Work Done' and 'Energy of Motion' and their mutual relation perfectly clear.

But the question how far the mathematical treatment shall be carried, and at what point the student must be called upon to accept results on the teacher's authority, is not one capable of a definite answer good for all occasions.

I have myself ventured (I am told) rather far, as the learners I had primarily in view are those under the guidance of a teacher, who can use a discretion, omitting the proofs and confining his class to an appreciation of results, when he thinks it necessary. But I have endeavoured to guide the solitary student, by throwing into foot-notes, or at least smaller type, such investigations as seemed both likely to alarm him and possible for him to pass over lightly. If ever an occasion occurs for revision of the work, this division may be carried farther, as experience may suggest.

These observations refer principally to my use of algebraical formulas, as in the sections on Connected Systems and Elastic Impact, or elsewhere. In the sections on Varying Forces and on Potential Energy I have ventured on what to me appeared much the most formidable of my tasks—an elementary exposition of what, in truth, is the conception known to mathematicians as the characteristic of 'Forces with a Potential.' And yet I do not think it can be shirked: the conception of 'Energy of Position' essentially depends upon it. Experience must decide whether I have been successful.

Passing on from this old doctrine to the modern discoveries which complete the theory, I have based the extended Proposition entirely on Experiment; and I have endeavoured to describe such leading facts and experiments (or at least their nature and principles) as may suffice to establish the equivalence between Dynamical Energy and changes in Thermal, Physical, and Chemical Condition. Here the difficulty is, in these days of rapidly progressing knowledge, to keep clear of erroneous and obsolete theories, and to give details true *as far as they go*; so that, in the words of Professor Huxley, the student 'may learn, with a fair prospect of having but little to unlearn.'

Lastly, I have endeavoured to give the young student some conception of the *possibility* of explaining the Correlation of Energies, thus experimentally established, by the theory that all phenomenal changes are really, in themselves, changes of motion and position among the molecules or ultimate atoms of substances. I hope I have succeeded in presenting this section distinctly as exhibiting a probable surmise, which may be false without vitiating the doctrine previously developed.

D. D. HEATH.

KITLANDS, *November* 1, 1874.

THE DOCTRINE
OF
ENERGY.

INTRODUCTORY.

THE full meaning of any term expressing a physical conception can only be apprehended after some familiarity with the class of phenomena with which it is concerned. In order, however, to give the student a general notion of the subject of this treatise, we may define Energy to be—

'The Capacity, or Power, of any body or system of bodies, when in a given condition, to do a certain measurable quantity of work'; that is, to change its own condition and that of other bodies, exhausting its power by the using of it.

Some familiar examples will illustrate this:

(*a.*) A clock, when wound up, has energy, by the lowering of its weight, to give its parts motion, overcoming their friction, &c., *until it has run down.*

The clock will not start, perhaps, without help—without a tap given to the pendulum; but the capacity, the *amount* of change that can be produced, does not depend on this, but clearly on the *amount of winding.*

(*b.*) A coal, burning in the air, has energy to convert itself into the *products of combustion,* water, carbonic acid gas, &c., producing *heat* during the operation.

Here, again, the coal will not begin its action without help—a lighted match, for instance. But the match burns itself out, and the coal burns itself out; and the energy of the latter is, *cæteris paribus*, proportional to the quantity of coal.

(*c.*) A cannon-ball in motion has energy, if so directed, to pierce a target, or to raise itself in the air, &c., greater the greater the initial velocity; but the exact relation between its initial state and total capacity for work is not so obvious.

We can see clearly enough that the burning coal and the cannon-ball impinging on an obstacle are producing change in the condition of surrounding objects, while their own 'energy' is thus being spent: the heated room and the broken target exhibit 'external work' as the result. And we shall see hereafter that this is true also of the clock running down, and in all cases of uncompensated exhaustion of power in the original agent.

But internal change may be conspicuously going on in a body or system of bodies, while external effect is quite or almost inappreciable.

Thus a pendulum set swinging in the exhausted receiver of an air-pump (with due precautions taken in the suspension) will go on oscillating for days perhaps, and without perceptible flagging for some thousands of oscillations together. And we can imagine, though not actually produce, a 'perfect vacuum,' and an arrangement for support or suspension, without friction or stiffness; under which circumstances this action may be supposed to continue for ever. The Planetary Motions are another more nearly perfect instance. The Earth, for instance, changes its distance from the Sun, alternately approaching and receding from it, with no slackening that we have as yet been able to detect by the nicest observations.

It must, however, be carefully noted that in every such case this 'internal work' is *alternate* in character. The bob of the pendulum is always changing its condition, passing from one extreme, characterised by the *maximum of elevation* with *no motion*, at the moment when it is passing from its upward to its downward course, to the other extreme of *maximum motion* at the *lowest point* the bob can reach. And in all intermediate stages it has *more* or *less* of motion, according as it has *less* or *more* of elevation. We may, then, distinguish between the 'capacity for raising itself,' which it has in virtue of its state of motion (like that of the cannon-ball), and the 'capacity for giving itself motion,' which it owes to its elevation (as the clock can run itself down); and we may describe this continual action as consisting in the *alternate* exhaustion and renewal of each of these *two kinds* of Energy.

With this explanation, then, it is equally true of 'internal' action—that is, of the relative and mutual changes of the parts of what we consider as *one system*—as of 'external,' where the action may extend over an indefinite range; that (so far as our examples go) each and every energy, every capacity or power of work, exhausts itself as it does its proper work.

Now the Physical Truth which this treatise is intended to expound is this:—

Every kind of change which takes place in bodies and systems of bodies, whether due to their internal or to their mutual actions, can be represented *numerically* by reference to one certain *standard unit* of change, or work done, in such wise that the result of the passage of any system from one condition to another may be calculated by mere additions and subtractions, even when we do not know how the change came about. And if such result be *positive* —that is, more additive than subtractive—the system has *gained* so much total energy (or power to do work of some kind) at the expense of some other system; if *negative*—that is, more subtractive than additive—then it has, so to say, parted with so much energy and handed it over to some other system. A change which adds energy to the system is 'work done upon it'; when the energy is found to have been diminished it has 'done work' on some other system. Thus, winding up the clock is doing work upon it, exhausting our own muscular energy in the operation: giving motion to the cannon-ball exhausts the power of the gunpowder. All this will become clearer as we proceed.

The principal changes which occur in inorganic nature—that is, in everything around us not referable to animal or vegetable life—may be, conveniently for our purpose, roughly classed as:—

(*a*) Motions and changes of position of bodies; either as of *wholes* retaining the relative position of their parts, or among their component parts. These may be called *Dynamical Changes.*

(*b*) Changes of temperature: or, *Thermal Change.*

(*c*) Changes in the internal condition of the body; as expansion, change from solid to liquid, or liquid to gaseous—which are, in a narrow sense of the word, called *Changes of Physical Condition.*

(*d*) Changes of Electric Condition.

(*e*) Changes in the way of Composition and Decomposition of the *Chemical elements*, of which all bodies are made up; or *Chemical Change.*

When we speak hereafter of Dynamical, Thermal, Physical (or internal), Electric, and Chemical Energy, the student will understand that, in general, we refer to capacities for work of some kind which the bodies or systems possess *in virtue of their condition* as to motion and position, temperature, &c., respectively. For example, a piece of coal may be in motion, and so act dynamically, like the cannon-ball; this would be Dynamical Energy. But it can also burn, and in burning may, as we shall see, do various kinds of work—heat a room, or convert ice into water, &c.: whatever the nature of the *work done* in burning the *energy* is chemical.

We will now take a typical series of changes, each engendered by its predecessor, and explain the application of our proposition to the whole series.

Imagine then:—

I. A quantity of water at rest in an elevated reservoir with a sluice-gate.

II. The sluice opened, allowing the water to fall (vertically, or in a sloping channel), gaining velocity as it loses height.

III. The falling water encountering a water-wheel, whereby it loses some of its own motion, and imparts motion to the wheel and machinery connected therewith.

IV. Heat produced by collision or friction, both where the water clashes on the wheel, and wherever there are rubbing surfaces; and most sensibly at the grinding surfaces of the mill-stones.

V. The mill-stones, working in a chamber of water, heating it up to 'boiling point.'

VI. When this point is reached, the water will not rise any more in temperature, but will gradually pass into steam.

VII. The steam so generated works a steam-engine.

VIII. Which works an electric machine:—that is, it turns a glass cylinder which, as it turns, rubs on one side against a properly prepared cushion (the 'rubber'), and on the other passes close in front of some metallic spikes connected with a large metallic surface (the 'prime conductor'). Heat is produced, as in all cases of rubbing; but, besides this, the prime conductor becomes 'electrically charged,' that is, it will exhibit all the ordinary properties of an electrified body.

IX. If a metallic knob project from the conductor and the hand be held near it, a spark of fire will flash between the knob and the hand, and a shock be felt. If, instead of this, a thin wire be carried from the conductor to moist earth, the wire may become sensibly heated. In either case, the conductor is *discharged*, to be charged again by the action of the steam-engine.

X. If the discharging wire, or a wet thread, is carried, with a slight break, through a drop of water mixed with a little sulphuric acid, on its way to the earth, it will show less heat, but will to a small extent decompose the water into its two *component elements*, the gases oxygen and hydrogen.

XI. This decomposition will be so small in amount in any practicable case, that the series would close here. But we can imagine the experiment performed on so gigantic a scale as to make it sensible; or another process, of a less familiar kind, may be adopted, in which no ordinary friction (as of the rubber and glass) occurs, and therefore the purely electric effect is greater.*

XII. The oxygen and hydrogen thus set free may be recomposed—that is, the hydrogen gas *burned* in the oxygen—and steam again produced, which might work a pump, and so help to replenish the reservoir, the emptying of which has been the cause of all these changes, which will all cease when the reservoir is emptied, or the sluice-gate shut.

Our proposition then is, that we may represent numerically (1) the power to produce changes which the water in the reservoir has in virtue of its quantity and its height above the mill-wheel: (2) how much of the power due to motion the water gains as it loses height in falling: (3) how much in motion, and how much in heat, it gives to itself and to the mill-wheel when it clashes on it: (4) how much heat goes to turn water into steam: (5) what the steam loses in working the steam-engine: (6) how much the steam-engine is retarded by working the electric machine and producing heat and decomposing the water.

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* The picture is from Jenkin's 'Electricity and Magnetism,' p. 281; but a detailed account would not be intelligible except to those familiar with the science. The steam-engine is applied to give a rotation to the two

And further, that the sum of all the effects so produced by the falling of a given quantity of water down the given channel will be equal to the loss of energy, or capacity to

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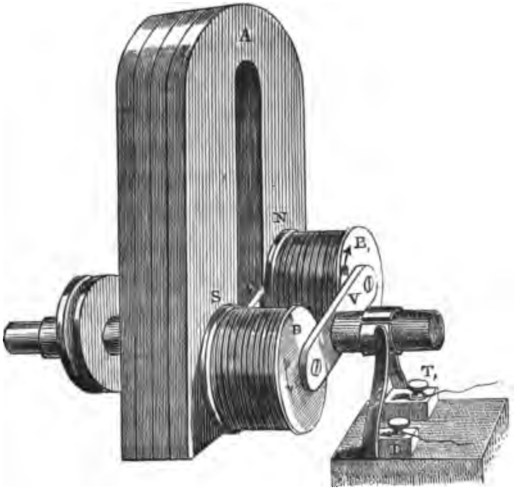
bobbins, B and B_1, round which is coiled an *insulated* wire (covered with silk, or it might be gutta-percha, as in submarine telegraphic cables), and inside of which are cores of soft iron. This apparatus revolves in front of a horse-shoe magnet; and, though there is no contact or friction, a stiff resistance is experienced, and the coils, &c., become heated. And if now the two ends of the wire, seen at T T_1, are carried into the cup of water, the decomposition will be considerable.



continue to produce such changes, caused in the reservoir of water by its gradual emptying.

Note.—If nothing were lost or otherwise disposed of on the way, and we could hand over to the final pump all the energy taken from the water in the reservoir, it would just restore the water to the reservoir at the same rate at which it parts with it; and there would thus be 'perpetual motion' kept up in the complete system,—the same ideal state as in our imaginary pendulum.

But this is, in fact, impossible. Heat is necessarily produced all through the machinery where there is clashing or rubbing, which goes off to surrounding bodies and contributes nothing to boiling the water and working the steam-engine. And, even if we suppose all clashing of unequal motions avoided, and all surfaces 'perfectly smooth,' and no radiation of heat from the surface of the vessel containing the grindstones, &c.—all which conditions may be more or less *approximated* to, though they cannot be attained—still the working of the steam-engine and of the electric machine necessarily and essentially (not as it were accidentally) requires the production of heat that *must* be thrown away (not destroyed, but taken out of the *circuit* of operations we are considering); in the one the heated steam which works the piston must be either cooled in the condenser, or suffered to escape into the air; and in the other, the friction of the glass and rubber must heat their surfaces as well as produce electrical charge. Some considerable part, therefore, of the original energy of the system, which we may consider as stored up in the reservoir, must leave the system in the form of heat, and does not contribute to keep the system going perpetually.

And it is worth observing, too, that even if we could get what is called a perfectly 'conservative system'—one of which our pendulum would be the simplest form, if we could absolutely get rid of friction and air—it would be only *self-sufficing*; could *do nothing*, could produce neither heat nor any other action beyond its own system.

Having thus expounded the general meaning of the terms used, and the fundamental propositions of the doctrine of Energy, we must, before proceeding to give the proofs of its truth, obtain clear conceptions of the primary physical quantities, and establish our standards of measurement of Matter, Force, Heat, and Work.

FUNDAMENTAL UNITS.

To measure any quantity is to find its proportion to some known quantity of the same kind.

We can recognise *sameness*, or equality in point of quantity, by a simple judgment of our sensations—as, for example, that two sheets of white paper are *equally* illuminated, or two trumpet-blasts equally loud: or we can assure ourselves that one sheet is brighter, or one sound louder, than the other. But we can only directly measure *numbers* of identical things and *proportions* of space and time, or, perhaps, more accurately speaking, of space only. We can, for instance, distinctly understand and verify the assertion that this line is twice as long as that, or that this volume is one-third of that; or that this body moves twice as fast as that, if it moves over twice the distance in the same time. But we cannot attach a clear meaning to the statement that this surface is twice as bright as that, except by first agreeing to measure quantity of light by reference to the number of candles, or to some effects which will be found ultimately to resolve themselves into measurement of space or time.

Our business, then, is so to form and define our conceptions of the fundamental changes in nature, dynamical, thermal, &c. (so far as we are concerned with them in this exposition), as to make them amenable to this mode of comparison.

MASS AND FORCE.

When using the language of science we mean the same thing, whether we speak of the 'mass' of any body, or of the 'quantity of matter' in it.

Now, so long as we are only dealing with one particular kind of matter—say the metal platinum—there is nothing abstruse about the meaning of the latter phrase, or the method we should adopt for measuring it. We should take, arbitrarily, some known portion of platinum as our *unit of mass*: which is what our Legislature has done, defining as the 'standard avoirdupois pound' a certain lump of platinum, which is kept in a Government office. Then the quantity of matter in any other piece of platinum will be so many times, or such a portion of a pound as its 'volume,' or the space it occupies, is of the space occupied by the unit. Only, as we know that the same quantity of platinum expands and contracts, however slightly, under different circumstances, we must, if we would be quite accurate, before making our measurements, take care to reduce the masses meant to be compared to a fixed standard of temperature, and of all other conditions which are found to alter the volume of the material.

But we cannot even attach any meaning to the question, 'How much iron or water contains as much matter as the 1 lb. of platinum?' without selecting, as the term of comparison, some measurable quality common to all substances, which reduces itself to mere volume when only one kind in one condition (as the platinum) is considered.

The most natural test, according to the popular use of language, may be *weight*, and we might say that all quantities of different materials which balance the 1 lb. of platinum on the scales *shall be called* pounds also, and be said to have the same mass.

This is, in fact, the practical test usually applied, and it is justifiable, and leads to no confusion, *because experiment shows* that changes of shape, temperature, &c. make no difference in the weight, and that masses that balance each other at one place do so everywhere else.

But mass is not *the same thing* as weight.

Weight is the strength of the downward tendency of any body, and may be tested by the extent to which it will press down a given spring which is made to support it. And this, for the same quantity of matter (platinum or other), varies in different places, at tops of mountains, or down in mines, or in different latitudes. Could we take 1 lb. up 4,000 miles high, and find what spring it would there depress by one inch, it would require four such springs to support it down here.

In truth, when we weigh iron, &c. against platinum, we are comparing their efficiency, under precisely similar circumstances, to produce motion or change of position in the scales. And the full scientific test and definition of 'equality of mass' is 'identical behaviour in respect of *forces*, or causes of motion.' We must therefore combine the study of the measurement of Force with this extended meaning of the word Mass.

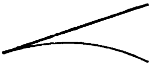
All notions about force and motion were in almost utter confusion until the discovery of the 'laws of motion,' more especially of the First Law, which was not only unknown till the times of Galileo, but may be said to be the very contrary of what was before then believed. It may be thus stated:—

The *rate* and the *direction* of motion with and in which (respectively) a body is moving at any moment is to be considered as part of its *actual condition* at that moment, which it will retain until some adequate cause changes either the 'velocity' (rate), or the direction, or both. We may reasonably enquire *how it got* the motion it has, as we may how it came by its shape, or its temperature, &c., and, again, under what circumstances it will change any of these properties; but not *why*, having got them, it keeps them.

Thus, if a sliding stone goes slower and slower, we are to look for the cause and law of retardation in something outside, and we find it in the friction of the ground and the resistance of the air. As these diminish the retardation of the motion goes on with less and less rapidity. So, when we find a planet continually changing the direction of its motion, circling round the sun, we find the degree of curvature, as well as the change of velocity, to be dependent on the position and distance of the sun.

There is no motion within our atmosphere, and, perhaps, none in nature anywhere, unaccompanied by friction and resistance; and we certainly know of none uninfluenced by causes *tending* to alter it; for all portions of matter seem to act and react on each other. But these mutual influences can counteract one another, as when we place a body on a flat table to counteract gravity; and we can almost indefinitely diminish friction and resistance of air; and we can imagine, at least as an ideal case on which we may reason, 'perfectly smooth surfaces' and 'perfect vacuum,' under which conditions the First Law of Motion only would give the rule;—in other words, the motion would be uniform and straight so long as these conditions continued.

Note.—There is no need to explain the meaning of 'direction' or 'velocity' when a body is moving uniformly in a straight line. When the motion is in a curve, we mean by the 'direction at a given point' of the curve the direction of the straight line which lies nearer to the curve at that point than any other: what in geometry is called the



tangent to the curve. And, in like manner, by the 'velocity at a given time' we mean the uniform velocity which is deviating less from the motion of the body at that time than any other. And the First Law tells us that if, in the first of the examples we have given, the rough surface were suddenly to become perfectly smooth; or if in the second the sun were suddenly annihilated; then the stone or the planet, as the case might be, would go on moving with the velocity and direction belonging to them respectively at the instant, defined in this way.

When, then, any body undergoes change in the velocity or direction of its motion, we have to look for the cause in surrounding circumstances; and these circumstances—in other words, the positions and conditions of other bodies—are said to 'exert force' on it; or it is 'subject to the action of Force.'

No doubt the original sense of this and equivalent words was connected with the *sensations* we have when pulling, pushing, throwing, or arresting a mass; in short, *straining* ourselves. And its use may probably have been first extended to cases where the influence or action of inanimate bodies most reminds us of our own physical condition when thus exerting ourselves: to trees bent by, and while bent resisting the wind; to stones pushed onwards by streams; to stretched or bent springs recoiling, &c. But we cannot reduce to measure the intensity of sensations. And moreover the influences of body on body in nature cannot always be—notably, the most universal of all these influences, the force of gravitation, has not as yet been—shown to be at all analogous to these pressures, strains, jerks, &c.

Our scientific *measure* of Force, therefore, must not be founded on any conjectured state of *strain* or the like, subsisting (say) in the space between the sun and the earth—although the case we shall take as our type or example of force is one of this nature—but solely on the observed effects—the changes of motion that ensue.

Place the unit of mass—the 1 lb. of platinum—on a perfectly smooth horizontal table; it will have no tendency to move. If a string be attached to it and pulled horizontally, it will give it motion in the direction it is pulled; and if, after a time, the string be cut, it will (First Law) leave the mass moving in the direction and with the velocity which it had acquired at the instant of the severance. The string has 'exerted force' on the mass.

Now, if the string be elastic it will be stretched while exerting force; and, if we suppose the string unaltered in its qualities throughout our experiments, the force it is exerting may be said to be the same whenever the *stretch of the string* (that is, the addition to its length) is the same, however the stretching may be caused; as by manual force, by hanging a weight to the other end over a pulley, by the attraction of a magnet on a knob of iron attached to that end, &c. The greater the stretch the greater the force, but not necessarily in proportion to the stretch.*

We have supposed our string visibly elastic; but in truth all strings and other materials do alter in length when pulled, however slightly; and it is by their 'elasticity,' or tendency to return to their unconstrained condition, that they become the instruments of transmitting—so to say—the pull of the hand, &c. to the mass meant to be moved. In truth, the string, in virtue of its stretch, exerts the force on the body; we exert force on the string to stretch it.

Note.—This is the principle of the *Dynamometer*, which may be seen in use in trials of ploughs and other implements or machines. A spring is interposed between the horse and the plough, and a comparison of the effect upon it in the way of unbending, or stretching out the coil, or otherwise, when different ploughs are dragged through the same soil, shows their comparative merits in respect of draught.

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* We are here talking of a particular string. When speaking of different strings, 'the stretch' means the proportion of the extension to the original length.

Definition.—If when the string, kept at uniform stretch, and in one direction, acts on unit of mass for one second of time, and is then cut, the mass is found to be moving at the rate of one foot per second (which is the English standard unit of velocity)—then the string, whenever and under whatever circumstances it is found *equally stretched*, is said to be exerting the unit of Force, or Force (1). And any other arrangement of surrounding circumstances which produces the same effect on the mass in the same time is similarly said to exert Force (1) upon it.

Place now m units of mass side by side on the table, and attach to each a string exerting this force, all parallel in direction. The m masses will begin to move in parallel directions, and at the same rate; and no alteration will be made in their motion if we suppose them all glued together while thus moving side by side. Therefore m units of force, or a force (m), is required to give to a mass (m) in 1^s the same velocity that force (1) gives to mass (1).

We might have taken any other sized lump of platinum and carried on our experiments for any other length of time and distance, and have got to the same *relative* conclusions. So that we may generalise, and say that 'Forces which, acting for *the same time* on *different* masses, give them *the same velocities*, are proportional to the masses they respectively act upon.'

And we might have used any other material—iron, water in a vessel, &c.—instead of platinum, and have come to similar conclusions.

And so we have obtained the common measurable property of matter which we are in search of, and may *define* Mass generally by saying that 'the unit of mass is that quantity of *any* material to which force (1) gives velocity (1) in time (1)'; and that, generally, 'the mass of any body is proportional to the *force* which, so acting, gives it velocity (1).'

As has been already mentioned, the expansion or contraction of the volume of any material, which commonly takes place with varying temperature or pressure, is found to make no dynamical difference; that is, the stretch of string required to give it a determinate velocity is unaffected. If therefore we use the word 'Density' to denote the *inverse ratio* of the volume of the unit of mass of any material in any condition of temperature and pressure to that of the unit of platinum in its standard condition, we shall have Mass = Volume × Density; and this agrees with the original meaning of the word, as used in respect of any particular compressible material.

Note.—Alteration of shape, or of the place of attachment of the string, may produce effects of *rotation* on the body, but would leave all that is here said as to the rate of progression of the whole equally true. The investigation of this subject belongs to the higher mathematics, but is based on and entirely consistent with the elementary principles here laid down.

We have now obtained numerical determinations or measures of Mass and Force, and might proceed to Heat. But it may be better here to complete our elementary conceptions of Force, in its relation to varied velocities and directions as well as to Mass, before leaving the subject.

SECOND LAW OF MOTION.

THE effect of a given Force acting on a given mass for a given time and in a given *direction* (that is, always representable by a string stretched in a direction parallel to and concurrent with its original line of action) is the same (estimated in that direction), whether the mass be originally at rest or in motion, or be acted on or not by other forces at the same time.

To make this intelligible, we will proceed from simple to complex cases. And, first, when the motion and the force are in one line.

(a) Let a mass be moving (under Law 1) in a straight line with an uniform velocity (V); so that in every period of t^s it will move through Vt feet. Then let a string act upon it for a time t, in the same line in which it is moving, and be kept, by whatever means, at a *constant stretch such* as, had the body been at rest, would in that time have caused it to move through a distance (s), and have left it moving with a velocity (v).

Our Law says that, if the force be *concurrent* with the original motion (that is, acting so as to give motion in the same direction), the body will, at the end of the time, have actually passed over the distance $Vt+s$, and be left moving with the velocity $V+v$; and if the force is *opposing* the motion (acting in the opposite direction), then the respective distances and velocity will be $Vt-s$ and $V-v$.

In this last case, if the force act long enough, it will end by overcoming the motion in the original direction, and impressing an opposite one on the body; and this will in time bring it back to, and then carry it beyond the starting-point. It is easy to see that after the backward motion has begun the velocity in the reversed direction will be $v-V$; and that after the body has passed its original starting point, the distance passed over in the reverse direction will be $s-Vt$. According to the usual convention about the meaning of the negative sign in the application of algebraical symbols to geometry, the formulas, as above given, will still apply: $Vt-s$ when s is greater than Vt meaning $s-Vt$ reckoned backwards; and similarly of $v-V$.

(b) If several strings, at different constant stretches, be acting simultaneously, some concurrent with and some opposing the original motion, then, in like manner, the space moved over (measured in the original direction) will be found by adding to Vt the distances by which each concurrent force would have carried the body onwards, and subtracting from the sum the distances due to the opposing forces, and so of the velocity.*

Note.—The principles established in (*a*) and (*b*) enable us to generalise our conception and measurement of Force, and to state numerically the result of the action of any force (F) acting for any time (t) on any mass (m), when the motion is in one straight line. Thus:—

By Force (F) we mean, according to our definition, any force equivalent to that exerted by F strings each stretched so as to be competent to produce velocity (1) in 1^s on mass (1) supposed at rest: and it will, we know, give velocity (1) to mass (F) (p. 16). We now know that these F strings acting *simultaneously* on mass (1), *concurrently with its actual motion*, will in the same time 'accelerate' (that is, increase) its velocity by (F).

So that the numerical measure of a force may be, indifferently, defined either by the mass to which it will in 1^s add velocity (1); or by the velocity it will add to mass (1) in that time.

Again, to produce velocity (F) on mass (m) will require force ($m F$); and force ($m F$) may be treated as the sum of m forces each equal to F. As, then, these m equal forces produce velocity F, each acting separately would (by our law) produce velocity $\frac{F}{m}$ (that is $\frac{1}{m}$th part of the effect).

That is, velocity produced in 1^s by force (F) on mass (m) is $\frac{F}{m}$.

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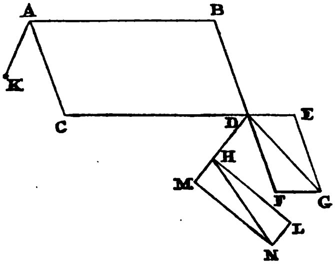
* The connection between (s) and the force and time will be shown farther on.

And as much will be added in each second that it acts, so that if v be the velocity acquired in time (t), $v=\frac{Ft}{m}$, or $mv=Ft$.

The quantity ($m\ v$) is what Newton called the 'quantity of motion' in the mass m when moving with velocity v, taking the motion in the unit of mass moving with unit of velocity as the *unit of motion.* Modern writers call it the 'Momentum.' And we see that a force (F) acting for a time (t) on *any mass* produces a momentum Ft; and that if a mass (m) be moving with velocity (v), then it may have attained that condition by the action of any given force whatever (F) acting on it for a time $\frac{mv}{F}$, or may have attained it by a force acting for any given time whatever (t), if that force be taken of the magnitude $\frac{mv}{t}$. When a mass in motion comes very suddenly to rest, as in the case of collisions, a very large force F has acted for an imperceptibly small time.

We now proceed to the illustration of our Second Law in other cases.



(c) If besides the forces acting along the original line of motion, A B [as in (a) and (b)] a new force, represented by a string always kept stretched in a given direction, A C—that is, kept always *parallel* to A C—begins to act, our Law is to be understood as follows:—

Had the original forces and velocities subsisted *alone*, the body would, after a time, have arrived at some point B on the line A B, and have there a velocity v in the direction of the motion, the position and velocity being determined as above indicated: had the original motions and forces not existed and the body been at rest, it would in the same time, by virtue of the new force alone, have arrived at some point C, in A C, the line of action of this force, and have a velocity v' determined as above.

Then the actual position of the body at the end of the t^s will be at D, on the line B D drawn parallel and equal to A C: as if the body had consecutively moved, first for t^s under the first conditions only, from A to B, and then (those first conditions and motion entirely ceasing) had moved for t^s under the second condition only from B to D; or (which comes to the same thing) had first moved from A C under the second conditions, and then from C to D under the first.

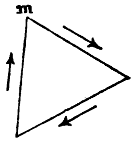
And the direction and velocity of the motion with which it will be left at D is determined on the same principles, thus:—

Had the first set of conditions alone existed it would have had a velocity v *in the direction of* D E (that is, in the continuation of A B parallel to D E): had the other force only acted, it would have a velocity v' in the direction D F parallel to A C. If D E and D F be the distances it would travel over uniformly in the next second under each condition separately, it will, according to the principles above explained, be found after 1^s at G, the farther angle of the parallelogram D E G F; and will have moved, and will continue to move, in the direction of the diagonal D G, with a velocity which is to v and to v' respectively as D G (the space actually moved over in 1^s) is to D E and D F.

(d) And so, if any number of forces be acting in different directions, the same rule applies. Thus, if a third force has been acting always parallel to the line A K, such as would, acting alone, have brought the body to K, with a velocity v'', the actual place will be at H, D H being drawn parallel and equal to A K; and the final velocity will be proportional to H N, the diagonal of the parallelogram formed by taking H L equal and parallel to D G (which represents the velocity due to the first two sets of conditions, as above explained), and H M, the distance which could have been travelled over in 1^s in virtue of the third force.

Corollary.—If two forces F and F' act on a body in two directions, such as D E and D F, and D E and D F are taken proportional to their magnitudes, the effects of each will always continue proportional, and the motion will be the same as that of a single force acting in the direction of the diagonal D G, and proportional to it in magnitude.

Note.—We began with measuring the force acting on a mass by the effect it produces on its velocity. But the study of the Second Law of Motion shows us that we may without incongruity extend our conception of Force, so as to consider a mass to be under the influence of forces where no change of motion is taking place, measuring the forces by the effects which *would follow*, if the circumstances were partially altered—that is (if we suppose the forces to be represented by stretched strings), by the effect which each string, at its existing stretch, would produce if all the others ceased to act. Thus, if three forces are acting on a mass in several directions, parallel and proportional in magnitude to the three sides of a triangle taken in consecutive order, it is easily seen that, on the principles above explained, the resultant effect is zero; and no motion ensues.



And so, more generally, when a mass alters the velocity and direction of its motion, it is indifferent, as far as the result is concerned, whether we represent the action by one string stretched to the extent and in the direction which would produce the actual motion; or by any number of strings stretched in different directions, whose combined action, calculated as above explained, would be equivalent to that of the single string.

The one force which represents the combined action of several is called their 'Resultant;' the two or more whose combined action is represented by that of one are called its 'Components.' To 'resolve' a force into others in given directions is to find forces in those directions whose combined action is equivalent to it.

Note.—When we are only considering the effects of several forces on the same body, it is convenient to take this body, temporarily, as the unit of mass, and measure the forces by their power of accelerating it per second of time. This is called the 'accelerative measure' of the forces in reference to this body, and a force so temporarily measured is called an 'accelerative force;' and, in contradistinction, the measure by reference to the standard unit is called the 'moving force.'

Now, it has been shown by the experiments of Galileo and others, that all bodies, large or small, heavy or light, fall to the ground from rest at the same rate, when the resistance of the air is insensible or got rid of; though that rate varies a little (as has been said) at different places. That is to say, the 'accelerative force' of gravity is the same on all masses; and therefore its 'moving force' on any mass is proportional to that mass. This experimental fact it is which entitles us to ascertain the mass of every body by weighing it (see pp. 11, 12).

It is usual to denote the force of gravity on the unit of mass by the letter g, which therefore varies somewhat according to the place, but is always a little more than 32 on or near the surface of the earth; and g is, therefore, also the *accelerative force of gravity.*

THIRD LAW OF MOTION.

Action and reaction are equal and opposite. That is, whenever one body is found exerting a force on another, that other is exerting an equal force in the same line, but in the opposite direction, on the first. As equal forces acting on different masses for the same time produce velocities inversely proportional to the masses (or, which is the same thing, produce equal *momenta*), it follows that, if motion ensue, the larger mass will move more slowly. (See p. 20.)

An example or two will bring out all the meanings of this law.

A magnet attracts iron, and the iron attracts the magnet equally. If each be placed on a separate piece of wood, and they be floated on water, they will be seen to approach; the larger mass moving slowly, the smaller faster. If they neither of them had any motion impressed upon them at first starting, when they come together they will *come to rest*, each pressing equally towards the other and destroying, by this opposing force, the momentum which the other had. If one mass had been started with a velocity, and therefore a corresponding momentum, this momentum must remain to the end in the whole system, because (the forces being at each instant equal and opposite) the new momenta which each produced during their action were equal and opposite, and so there has been no force to destroy the first impressed momentum. Therefore, the joined masses will go on in the direction of the first impressed motion, but at a slower rate, so as to keep *the same momentum*.*

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* If m and m' be the two masses, and v the velocity impressed on m, then mv is the first momentum. If u be the final of velocity of the two, $(m+m')u$ is the final momentum. These must be equal, and therefore

$$u = \frac{m}{m+m'}\,v.$$

The wood, at the point of contact, is in fact always compressed, however slightly, by the mutual pressure; and we may conceive the action better by imagining a spring, or other visibly elastic material, interposed. The magnet is pressing on its end of the spring to get at the iron; the spring is pressing against the magnet to regain its natural state; and these two forces are equal and opposite. So at the other end, between the iron and the spring. And, finally, the pressures on the spring in opposite directions at its two extremities must be the same, or one would prevail, and the whole apparatus move on.

It is to be observed that the usual way in which we speak of gravity or weight, as merely a tendency downwards, would seem to imply a force acting on the stone, &c., with no reaction. But the truth is, as Newton and others showed, that the stone exerts the same force on the earth that the earth does on the stone. But the velocity given to the earth is quite insensible, owing to its size.

Note.—The statement of the First Law of Motion, as bidding us treat the rate and direction of the motion of any body as part of its actual condition, seems necessarily to imply a reference of that motion to Space, conceived as absolutely fixed; and the effect we attribute to Force, and by which we profess to measure its intensity, is the change produced, not in apparent but in this absolute motion, by surrounding circumstances.

And yet we are directly sensible only of relative motions. If we are shut up in a ship's cabin, and the motion is perfectly uniform, our senses do not reveal its existence. And if we can look out of the cabin-window at distant objects, we can only say that *there is motion*:—whether of ourselves, of these objects, or of both combined, our eyes cannot tell.

But the student may see that, if we take the Laws of Motion to be true, we may reason from them, and apply conclusions to relative motion, while ignorant of what the actual motions are ; and, in many cases, may by their aid detect real motion in apparent rest, or resolve apparent motions into a combination of real ones.

Suppose, for instance, a vessel moving with absolutely uniform and rectilinear motion along a river. (We must conceive it as acted on by the wind or some other agent tending to impel it and accelerate its velocity, while the resistance of the water tends to retard it; and our supposition is that these two tendencies balance each other, and produce a resultant zero (see p. 22), leaving the First Law to govern the motion.)

If now we step from the shore on to the deck, supposed perfectly smooth, we shall apparently slip backwards ; in truth remaining at rest while the vessel moves forward under us. If the deck is sufficiently rough it will prevent this relative motion: we shall feel a shock and tendency to fall backwards ; and finally the force of friction will act upon us until it has equalised our velocity and that of the vessel ; after which we shall go on with it by the First Law under the influence of *no force whatever*: not because we are in the ship—for we should go on with the same motion if the ship were annihilated—but because the surface of the deck *did*, once for all, exert force upon us.

If now the vessel's speed be checked, we shall experience the reverse effect, and be apparently thrown forward. Or if, without alteration of speed, the vessel were to round a corner, the passengers would be forced by friction, with more or less of a jerk tending to throw them towards the outer edge, to alter their course also. And these bodily experiences would suffice, to a careful and well-trained observer, to indicate changes in the absolute motion of the vessel, without looking at all to the corresponding apparent and relative changes between ship and shore.

Again, suppose that while we and the ship are moving uniformly, under no force or balanced forces, we make the experiment with the smooth table, mass, and stretched string, with which we illustrated the nature of Force and Laws of Motion. We should be measuring effects by the apparent motions on the moving table, and thence draw our inferences as to mass, force, &c., as above, neglecting all consideration of the fact that table, string, and mass were moving independently of our actions. And our conclusions would, nevertheless, be correct. The strings, which we keep apparently parallel to their original direction, are really so: and the effect produced *by them* will, by the Second Law, be the apparent, not the total and absolute, motion. But if during the experiment the ship is from time to time altering its velocity or direction, our experiments will result in a jumble of confusion, unless we can make out what these alterations have been.

These simple examples will, it is hoped, be enough to illustrate this point, and remove difficulties which might occur to a thoughtful student new to the subject. He may consult Newton's Preface to the 'Principia' for more illustrations; and may hereafter learn how the diurnal rotation of the earth—which is analogous to our ship continually turning a corner—can be detected on these principles.

He may also be led to perceive that the unhesitating faith with which men trained in physical research accept the great laws of nature does not, in general, rest on the conviction brought by one or two special experiments, however carefully made, but on the way in which they are found to fall in with and harmonise observations of the most complicated facts. While the ancients believed that bodies, when no force is acting on them, either come to rest or move to some particular place appropriated to them in the universe, and distinguished between the motion of a body carried in a vessel and that of the vessel which carries it as essentially different phenomena, nothing but confusion could arise from the most acute reasoning based on these suppositions: the true laws make the reasoning easy and the results consistent.

MEASUREMENT OF HEAT.*

It will be seen from the typical series of actions set forth for our consideration, that we have principally to deal with the production of heat, and its effect on the physical condition and the motion of matter. But we take our measure from its simpler and more obvious action—that of passing from body to body according to their difference of *temperature* as shown by the thermometer. It may be as well to observe, for those who have heard or read nothing of modern research, that our thus speaking, as in familiar discourse, of the passage of heat from body to body, does not imply—what no one now believes—that it is a material thing. We speak as readily of one body imparting some of its motion to another when it impinges on it. And we shall see reasons for thinking that the transference of heat is something analogous to this. What is required, to justify our language and method, is to show that we can get such a definition and measure of the *quantity* of heat so passing as shall enable us to state consistently all the laws of phenomena of this kind.

We define, then, the unit of heat as that which passes to or from 1 lb. of water when it is raised from the temperature 60° Fahr. (*i.e.* on the common English scale, in which freezing-point is 32° and boiling-point 212°) to 61°, or reduced from 61° to 60°. The heat necessary to raise 2 lbs. of water thus much is two units, and so on. And we must show, or at least indicate, the kind of observations and experiments which will suffice to prove that this leads to no confusion.

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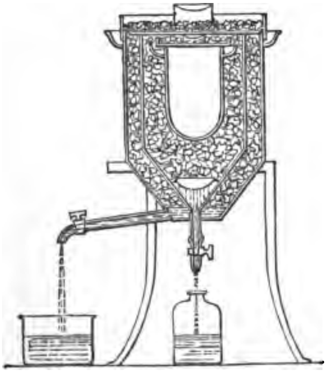
* Only what is necessary for our purpose is set down here. The student is referred to Professor Maxwell's or other treatises for a fuller exposition.

Suppose we take an urn-heater of known size, and heat it to an ascertained temperature, and then find that it exactly heats the water in the urn from 60° to 61° before it loses all its own excess of temperature and comes down to the common level of 60°. If now a quantity of some other material, say mercury, of a known lower temperature, be poured into the water and stirred in it, and the quantities be so adjusted that, on finishing the operation, the whole attains the common temperature of 60°—supposing such precautions taken that no perceptible amount of heat passes out of the vessel into surrounding space, or that due *allowance* can be made for what inevitably will so pass—then the water has imparted to the mercury, according to our definition, all the heat it received from the heater.

Therefore, for our method to hold good, if the heater be employed to heat the same quantity of mercury directly, instead of circuitously through the water, it ought to do so to exactly the same amount. And experiments *of this kind*—that is, involving the same principle and test—have been made,—and with this result.

It is found that the same quantity of heat, when applied in this way to heat different materials, does not raise equal masses of them through the same number of degrees. Water requires more heat than most substances: what will raise 1 lb. of water 1° will raise about 30 lb. of mercury or 9 lb. of iron by the same amount. Neither will the unit of heat raise 1 lb. of water by exactly 1° F. if it starts at any other temperature than the standard we have assumed of 60°; though the difference is small. There is, therefore, no such foretelling, from a few first principles, like the Laws of Motion, what other work the unit of heat can do, besides the work by which we have agreed to define it. Only it is necessary, for our method to be a proper one, that when we have experimentally formed a scale of *equivalents* of temperature and heat for all known materials at all points of the scale of temperature—that is, of the rise of temperature produced by the expenditure of an unit of heat upon the unit of mass of every substance at every temperature (as above of iron, mercury, and water at 60° F.)—they should be found *constant* and interchangeable in any *order* of operation: as was seen in the case of heating mercury, either directly by the heater or circuitously through the medium of the water.

But, in all such experiments, we must be careful that no *other work* is done besides heating. Thus, if we heat water in an open vessel, we not only do so, but simultaneously evaporate some of it. We must, then, either learn how to make allowance for this extra work or make our experiments in a closed vessel. Experiments and observations of the most various kinds have long been made, and this branch of the science of heat is fully established.



Note.—As we shall have occasion, in subsequent sections, to suppose quantities of heat measured, and it is desirable that the student should have some definite conception of how this is done, it may be as well here to describe one kind of 'calorimeter,' or apparatus for measuring quantity of heat, instead of referring him to other treatises. Although, strictly speaking, principles are involved and assumed which will have to be more distinctly proved farther on, yet they are so simple and familiar that no difficulty will be felt. These principles are that heat may be applied to melt ice, instead of warming water, and that, while that operation is going on, the ice retains one fixed temperature (32° F. or 0° Centigrade), and that a definite quantity of heat is always spent in melting a definite quantity of ice : viz. 144° *units of heat* to melt 1 lb. of ice at 32°.

In the figure the innermost chamber is intended to receive the body or material which is giving out the heat intended to be measured. This is surrounded by two chambers, communicating with two separate vessels, and each filled with broken ice, at melting-point (32° F.); and the room where the experiment is made is kept at some higher temperature. The chambers are parted by very thin sheets of good conducting metal, and we may, for our purpose of illustration, suppose that they simply let the heat pass from chamber to chamber without abstracting any sensible quantity.

Then the warmth of the room keeps the ice in the outer chamber at the fixed temperature, and so no heat passes inwards or outwards between it and the middle chamber, which is also at 32°. The heat from the material under experiment, therefore, alone passes out of the innermost to the middle chamber, and there produces no increase of temperature, but melts ice in proportion to its quantity. And this quantity is ascertained by measuring the quantity of water that is collected in the vessel under it.

It will be seen that neither evaporation nor rise of temperature is caused; and so the whole heat lost by the material is employed in the one work of melting ice, and the conditions required above for accuracy are completely satisfied.

MEASUREMENT OF WORK DONE.

The doctrine which is here meant to be explained, illustrated, and as far as may be proved, is, that we can compare numerically all the physical changes, such as have been mentioned, which depend one on another, and which are, therefore, said to be each 'the work of' the preceding condition of the system we are considering. The first step must be, to select one such change the most obviously capable of being estimated quantitatively; and we find this in what is, in the most popular sense, known as *doing work* and, at the same time, *exhausting power*.

Suppose we have to raise a given weight from the ground to a given height.

We may do this (whether by a direct pull or by a windlass) with a steady slow strain, starting with the slightest possible jerk, and landing it with just as slight a relaxation of upward pull; so that, beginning with rest, it ends at rest, and has had no considerable velocity at any time, and that the whole permanent change produced by our exertion is a *change of position*, brought about against the force of gravity. We know well enough, by our feelings, that this work tends to exhaust us; and investigation shows us that our nerves, blood, and tissues are chemically affected by it, and require restoration by a supply of food. The calculation of all this would be very complicated, and indeed uncertain, but it is obvious that the raising of two or more weights, or of the same weight to double or a greater height, may always be effected by an exactly *proportional* exertion—by two persons using two windlasses in the first case, for instance, and by continuing the winding twice as long in the second. So that, *if there be such a standard as we are seeking*, we may take this kind of work for it, as the simplest. And, accordingly, in practical engineering the *unit of work* is defined as—

The raising of 1 *lb. weight to the height of* 1 *ft.* And the raising of m lbs. weight to the height (h) will, on this scale, be $m\ h$.

For the present we suppose the operation performed, under the conditions above specified, by a very slow strain, beginning and ending with rest, so that any velocity communicated in the course of it is not only *transient*, but may be conceived to be throughout insensible, and therefore need not be noticed, as disturbing our conception of the operation. We shall see hereafter that it is not really necessary to make this supposition as regards the conclusion we are aiming at.

This unit is known as the 'foot-pound' unit. We may generalise and conceive work to be done against any other constant force, still under the same specified conditions: for example, dragging a plough on a level against the friction of the furrow. The force exerted may be tested by a dynamometer (p. 15).

Suppose it to be F, then the operation will be equivalent, so far as the exertion or expenditure of power goes, to that of lifting a mass $\frac{F}{g}$ against gravity to a height equal to the length of the furrow, and will be numerically $\frac{F}{g} \times$ *length of furrow*. When looked at as changing the condition of the weight and the plough respectively there is a great difference, which we shall have to consider hereafter.

In purely scientific treatises, in which the forces are measured as we have explained and defined (p. 15, &c.), it is more systematic, and therefore better, to take as the unit of work the pulling or pressing against the *unit of force* through the unit of distance. In this 'absolute' scale of measurement the work on the plough will be $F \times$ distance ploughed; the 'foot-pound' will be g; and the raising mass (m) to height (h) will be $g\,m\,h$. In this treatise, when 'foot-pounds' are not mentioned, this absolute scale is understood to be referred to.

DYNAMICAL ENERGY.

I. ENERGY OF MOTION.

Recurring now to our first instance, we may effect the very same *result*—of raising the weight from rest on the ground to rest (momentary rest, at all events) at a given height, by a very different kind of exertion—by a jerk of the arm, or the explosion of gunpowder just sufficient for the purpose; that is, by giving to it, *by any means whatever*, a proper velocity upwards, and then leaving it to itself—the case of the cannon-ball. In other words, the *possession of velocity*, howsoever acquired, gives to any mass an *intrinsic energy* capable of effectuating for itself the same change in its position, against the force of gravity, or any other force that may oppose its motion, which can be produced *upon it* by the application of the hand, &c.; the mode of action being different in the two cases put, but each agreeing in the fact that the working energy is spent as it works. We ought, then, to be able to find some such measure of the 'Energy of Motion' of any mass in motion as that one unit of the energy expressed on this scale shall give it a capacity for doing one unit of work, and so in proportion. And this we can do by the help of the principles we have already established.

Let a mass (m) be moving at any instant with velocity V, and let a force, F, act against it in the line of its motion for a time t; and let s be the distance moved over, and v be the velocity at the end of that time.

The force is here diminishing the motion, and the diminution in the time t is given (p. 19, &c.) by the equation

$$m(V - v) = Ft \ (a)$$

Again, as the force subtracts equal velocities in equal times throughout, the *average* velocity with which the body has moved during the time t is the *arithmetical* mean, or half-way, between the initial and final velocities; that is, $\frac{V+v}{2}$. And therefore the distance travelled, or $s, = \frac{V+v}{2} t \ (b)$. Eliminate t (by multiplying equations (a) and (b) together, and dividing by t), and we have

$$\frac{m V^2}{2} - \frac{m v^2}{2} = Fs \ (A).$$

If the action is allowed to go on until it has completely exhausted the original velocity and consequent energy; that is, until v becomes $= o$, we have for the distance (S) through which the mass m, with initial velocity V can force its way against the force F before it comes to rest, $\frac{m V^2}{2} = FS$.

Now, FS is the work done, measured on the absolute scale, when the body is moved through this distance, against this force, by an external agency which does not leave it with any momentum (p. 33). The quantity $\frac{m V^2}{2}$ therefore measures the capacity of the mass (m) with initial velocity (V) to effect the same change, and must be taken as the total 'Energy of Motion' which it possesses. It sometimes goes by the name of the 'vis-viva,' and it may be described as half of the product of the mass by the square of its velocity, or of the momentum by the velocity.

When the force acting is that of gravity its amount is $m g$, proportional to the mass on which it is acting (p. 23). In this case our formula becomes $\frac{V^2}{2} = gS$; and the height to which a cannon-ball shot up vertically will rise depends simply on its initial velocity, and $= \frac{V^2}{2g}$, which is therefore the foot-pound measure of energy of motion. But if, instead of this case, we take that of the cannon-ball penetrating a target, the resistance of the materials will depend, not on the mass, but on the size and shape of the ball—principally upon the size of its transverse section—and other circumstances. If we suppose this resistance to be the same at each point in the thickness of the target, whether the ball be moving fast or slow—that is, if we take it as a *constant force*—the penetration will be proportional to the energy, as we have measured it: a ball twice as heavy, and moving twice as fast, will penetrate eight times as far before coming to rest.

But this is not all that we can learn from equation (A). $\frac{m V^2}{2}$ is, we have seen, the Energy of Motion of the body in the position it has when first considered; and $\frac{m v^2}{2}$ is the Energy of Motion which it has after moving through the distance (s) against the force (F). The difference $\frac{m V^2}{2} - \frac{m v^2}{2}$ is, therefore, the Energy of Motion which it has expended, or lost, in forcing itself along this distance: and this we see, $= Fs$.

Now, this is precisely the amount of energy that is required simply to carry it through this distance against this force, starting it and leaving it with no motion in it. That is, the Energy of Motion necessary to be *spent* in carrying a body through a given distance against a given force is the same, whether the body starts with any superfluous energy or not; or, as is easily seen to follow from the nature of the reasoning, whether, while it is moving, it receives any accessions of such energy from external sources or not. And we may, therefore, suppress the conditions which we imposed upon ourselves in the first instance (pp. 32, 33), and in every case take as the measure of the work done against a force the product of that force by the distance moved through against it. If in the case just put we exert more force than is necessary for lifting the weight we shall, besides doing this work, also impart a velocity and a corresponding Energy of Motion to the mass, which it will retain when it has reached its new position, and which, if not counteracted, will carry it proportionally farther upwards.

Just as, where the mass is doing work against the force, by virtue of its Energy of Motion, the energy spent is measured by Fs, so it is easily seen (and might have been expected) that, when the motion is in the direction of, and so is helped on by, the force, the Energy of Motion is increased by Fs. For the increase of momentum is now $m\ (v - V) = Ft$; and the distance moved over is still $\frac{v+V}{2}\ t$. Whence equation (A) becomes $\frac{mv^2}{2} - \frac{mV^2}{2} = Fs$.

The force is in this case said to be 'doing work on the mass.' And we may combine these two results into one Proposition if we call the product Fs the work done 'by' or 'against the force'—or, by an equivalent phraseology, 'upon' or 'by the mass in motion'—according as the motion is concurrent with or opposed to the tendency of the force. We may then say that—

'The Energy of Motion is increased or diminished, as the case may be, by the amount of work done by or against the force; or (otherwise stated) upon or by the mass.'

Note.—In accordance with well-known mathematical usage these two corresponding but reversed actions are commonly distinguished as *Positive* and *Negative* work: the positive being usually understood to be work done by the force, and the negative that done by the mass in motion. On this understanding the equation $\frac{m\ v^2}{2} -$

$\frac{m\ V^2}{2} = Fs$ may be taken as comprising both cases, the positive or negative value of Fs, according as v is greater or less than V, indicating that work has been done by or against the force.*

If more than one force is acting in the line of motion, some with and some against the motion, it follows from the second law that the Energy of Motion will be increased by the sum of the work done by, and diminished by that done against, the several forces; the process of calculation being the same, in effect, as reducing all the forces to one resultant, the *difference* between those acting in one direction and the other, and applying our rule as above given.

If we look back to our original type of work done (p. 32), we see that it is a case of this kind. The hand, or other agent pulling the weight upwards, is exerting a force upwards sensibly equal to that of gravity downwards (as might be tested by a dynamometer). Consequently, the work done *by* it is sensibly equal to that done *against* gravity.

On the other hand, if the force is exerted on the mass with no other force opposing it—as where the weight falls freely, or a stretched string pulls a mass along a smooth table—the measure of the work done by the force is the Energy of Motion communicated to the mass. The mere change of position, in such a case, *costs nothing*; the velocity may be produced indifferently, either once for all, by a powerful jerk or explosion during an inappreciably short time (p. 20), and while no perceptible change of position has taken place; or by a steady pull through a considerable distance; or, again, by a series of intermittent pulls or jerks at successive stages of the motion. In each case the velocity and consequent energy once given will continue for ever unspent unless some opposing force is met with.

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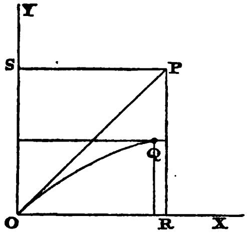
* The relation between the force, the time it has acted, and the space moved over by the mass, promised in the note at p. 19, is deduced from the above equations (a) and (b), and the corresponding ones for the case of force and motion concurrent, by eliminating v. We have $s = Vt - \frac{Ft^2}{2}$, or $s = Vt + \frac{Ft^2}{2}$. But we do not want these equations for our purpose here.

OBLIQUE MOTION.

THE Second Law enables us to carry this investigation on to the case where the force acts in a direction different from that of the motion, as when the cannon-ball is fired aslant. The student may keep this case in his mind as a familiar example, but the reasoning applies to any motion and constant force.

Let a mass (m) be at a point O, with a velocity (V) in the direction O P, and be acted on by a force (F) in the line O Y, and (we will suppose) downwards (in the direction from Y towards O). Draw O X perpendicular to O Y (that is, horizontal, if O Y be vertical). If we take any point P in O P, and drop the perpendiculars P R and P S, then the velocity V in a direction O P may be considered as *compounded* of two velocities, U and W, in the directions O Y and O X; the three quantities being proportional to the three lines, O P, O S, and O R (p. 22).



Now, O R P being a right-angled triangle, we have (Euclid I. 47) $OP^2 = PR^2 + OR^2 = OS^2 + PR^2$, whence

$$V^2 = U^2 + W^2$$

The Energy of Motion is $\frac{m\,V^2}{2}$, which is therefore equal to

$\frac{m\ U^2}{2} + \frac{m\ W^2}{2}$ which two quantities are the energies the mass would have in virtue of the two component velocities U and W taken singly. So that we may call $\frac{m\ U^2}{2}$ and $\frac{m\ W^2}{2}$ the *components* of the energy in the two directions, O Y and O X.*

Now (Second Law), the velocity W, and therefore the corresponding component of the energy, will remain throughout the motion unaffected by the action of the force F; and the effect of this force on the velocity and distance travelled parallel to O Y will be exactly the same as if W were non-existent. Therefore by our formulas, if the body comes in the course of its motion to any point (as Q in the figure), having risen (or fallen, as the case may be) by a distance s, the component $\frac{mU^2}{2}$, and therefore $\frac{m\ U^2}{2} + \frac{m\ W^2}{2}$ or $\frac{m\ V^2}{2}$ will be diminished or increased by Fs—or by the *work which would have been done* had it moved *directly* through the space s against the force.

We may, then, generalise, and define as 'work done against or by a force,' the product of the force by the distance moved through *estimated in its direction*—that is, the length intercepted on any line drawn parallel to that direction by two perpendiculars drawn from the first and last position of the moving mass, taken with the proper sign, according as the motion has been against or with the force. And we may still say that the energy of motion spent or gained is always measured by the work done.

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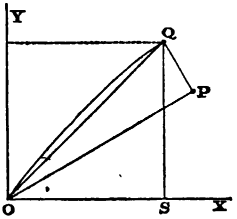
* Velocities and forces may be decomposed in directions making any angles we please with each other (see the figure and explanation, p. 21, &c.). But our reasoning here can only be applied to the decomposition of Energy of Motion in directions at right angles to each other.

If forces act simultaneously in the direction O Y and O X, they will each independently affect the correspondent component of the energy; and the total Energy of Motion will be changed by the sum or difference, as the case may be, of the work done in each direction.

Note.—The student may perceive that there are two ways in which we may ascertain the work done under the action of a force, which must, of course, lead to the same result.

If a mass move, by any path, from O to Q, having been acted on by a force F always in the direction O P, our reasoning shows that the work done is to be measured by F × O P.



But the force F may also be treated as decomposed into two forces (say X and Y) acting in any two lines O X and O Y; and if O X and O Y be taken at right angles to each other, we have also shown that each will do its work independently of the other. So that if Q S be dropped perpendicularly on to O X, O S and S Q will be the distances moved through in the directions of the forces X and Y, and the total work done will also be represented by X × O S + Y × S Q. These two determinations of the same work done must therefore necessarily come to the same thing.*

In like manner, if there be any number of forces acting simultaneously on the mass, to ascertain the work done we may either, first, compound all the forces into one resultant, and then multiply this resultant by the distance moved through in its direction; or we may proceed thus: let the mass move from O to Q, being acted on by forces F_1, F_2, &c., parallel to OF_1, OF_2, &c.

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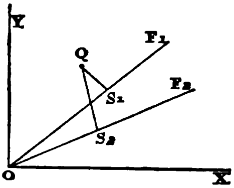
* A slight acquaintance with the formulas of trigonometry will show that this is so. For, join OQ, and call the angles POX and QOX, A and B. Then the components X and Y of the force F are F cos. A and F sin. A respectively; and OS = OQ cos. B, and SQ = OQ sin. B. Whence X × OS + Y × SQ = F × OQ (cos. A, cos. B + sin. A, sin. B), = (by a well-known formula), F × OQ cos. ($B-A$), or F × OQ. cos. QOP. And OQ cos. QOP = OP.

Whence, finally, X × OS + Y × SQ = F × OP. Q.E D.



Drop perpendiculars QS_1, QS_2, &c.; then OS_1, OS_2, &c. are the distances moved through in the directions of the several forces; and the work done by each force is $F_1 \times$ OS_2, $F_2 \times$ OS_2, &c.; and the total work done, and consequent increase of Energy of Motion, is the sum of all these, if all *do work* on the mass. If some act with and some against the motion, their effects must be added or subtracted accordingly, and the result, positive or negative, will give the increase or diminution of the Energy of Motion.

The proof would be by resolving each force, F_1, F_2, &c. into components along the same lines O X, O Y, at right angles to each other, and then re-compounding them, as in the last proposition.

The figures and reasoning suppose all the forces to be acting in the plane of the paper; but the slightest acquaintance with solid geometry will show that the result is true, in whatever directions the forces may be acting.

Note.—Every force may be represented by the action of a string stretched in the direction of the force. This suggests the conception of 'the point of application of a force,' and accordingly the work done by each force is often described as the 'product of the force by the distance travelled over by its point of application, estimated in its direction.'

Note 2.—It is important that the student should observe that, according to our definitions and reasoning therefrom, a force, when not acting in the line in which the mass is moving, produces a sensible effect, which, nevertheless, does not count as 'work done;' namely, curvature or alteration in the *direction* of the motion. We shall see more fully hereafter how this falls in with our general doctrine; but it may be at once shown that it is consistent with it, so far as we have got.

For 'work done' is a change which exhausts one capacity and correspondingly increases another; and it is obvious that a force producing curvature is thereby neither exhausting nor increasing the energy of motion, which depends only on the *velocity.* Thus, if two bodies be projected, one vertically and the other aslant, with the same *vertical* motion, all that gravity can do in the way of checking or increasing the capacity of either body is first to exhaust, and then to restore and increase in the reversed direction, the vertical part of the velocity, leaving the horizontal velocity of the second body untouched. The ultimate destruction of all motion, vertical and horizontal, when the bodies reach the rough surface of the ground (which is, of course, greater in the second case), is not caused by gravity at all, and will have to be considered separately.

In this Elementary Treatise our demonstrations are necessarily confined to cases which do not require the use of the higher mathematics; to such as are concerned with constant forces; and to some others where, though the forces are in fact complex, the result is simple.

But the student will have no difficulty in forming the conception of 'work done' against or by a varying force. For instance, if we raise water out of a well by a bucket and windlass, and the bucket leaks, and so gets lighter and lighter; or, if a cannon-ball penetrate a considerable way into a complex target, which gets weaker and weaker as it yields, the muscular energy or the motion of the cannon-ball will hold out the longer for the weakening of the resistance, but will, nevertheless, be spent gradually, as in simpler cases. So, on the other hand, if for the target we substitute a strong spring, when the progress of the ball has been arrested the spring will recoil and restore motion to the ball, with an effect gradually weakening, instead of constant; but, in other respects, it acts on the same principle on which the constant force of gravity will pull a ball down which has worked its way upwards against it.

And a general explanation or illustration may be given of the method of actually calculating the work done in such cases, when we know the *law* of the variation of the force—that is, how to find it for any and every stage of the action—without making too great a call on the student's powers.

Take, for instance, a cylinder of air—say 100 inches in height—closed by a movable piston; and let the piston be weighted or jerked downwards, so as to compress the air. The air will resist the motion. And if we take measures to get rid of any sensible change of temperature (which, as we shall hereafter see, would otherwise affect the action), the force of resistance will increase in proportion with the increase of *compression*; so that after the piston has moved through, say, 20 inches, the force of resistance will be increased by one-fifth of its original amount.

But the change, as the piston moves down to this point, is gradual; there is very little difference between the force at 100 and at 99 inches, or that at 90 and 89, &c. So that if we calculate according to the above law the actual force at 99, 98, 97 inches, &c., and take, as the work done while the piston moves through the 20 inches, the sum of the products of each of the calculated forces, at 100, 99, down to 81 inches, by 1 inch—in other words, the product of the *arithmetic mean* of all these forces by

20 inches; that is, by the whole distance moved through —we shall be pretty near the quantity we want. And we shall be nearer still if we divide the space into half-inches or quarter-inches, and so on, taking smaller and smaller distances before striking our mean.

And the higher mathematics give us the means of finding what the exact amount is, to which we shall, by such a process, gradually approach; or, when this cannot be done, it gives rules for carrying on the approximation as far as we please; just as we can, in common arithmetic, approximate to the value of surds.

We shall presently state some of the conclusions to which investigations of this nature lead. But we will first consider the other simple cases above alluded to, as not beyond our means of fully working out here.

CONNECTED SYSTEMS.

Bodies may move over the surfaces of other bodies, or may be connected together by stretched strings, or by pushing or pulling rods.

These connections, so far as they in anywise alter either the velocity or the direction of the motion of any mass from what it would otherwise have been, exert force upon it, according to the meaning we have attached to the word; and we may imagine, in each case, the actual surface, rod, &c. removed, and its place supplied by a string attached to the mass, kept, at each instant, to such a stretch and in such a direction as will produce the same effect.

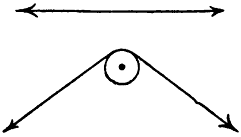
(a) We have already used for illustration the ideal conception of a 'perfectly smooth,' to which we ought to have added the adjective 'hard' surface.

We may define what this means more precisely by saying that it is a surface whose action can be perfectly represented by a string kept always perpendicular to the surface, and always stretched to the exact degree that will, at the instant, just balance and neutralise the action of all the other forces tending to do work against the pull of the string.

There does not, in fact, exist such a surface: every natural surface both tends to prevent motion along it and also yields, however slightly, to pressure—that is, to any force which may be exerted on it perpendicularly. But we can approach to the ideal; and it not only simplifies elementary calculations to suppose the perfect state, but we shall see that a consideration of the falling short—so to say—of nature from this ideal, forms a most important part of our subject.

(b) By a 'perfectly flexible string' we mean one that offers no resistance to any force, or the resolved part of any force, acting on it at right angles to the direction of its length at the point of application of the force. It can therefore be bent round any surface (as of a pulley or wheel) with no expenditure of energy, except what is necessary for giving it so much *motion*, in proportion to its mass.

When a string is thus coiled round a perfectly smooth surface and stretched by forces at its extremities, the action of the surface makes no difference in the stretching of the string; that is, it does no work on it—neither *along* the string (which is also along the surface) because, the surface is perfectly smooth, nor perpendicular to the surface, because the string is perfectly flexible. The action and reaction, therefore, of the forces at the extremities of the string remain equal and opposite, and produce their effect in stretching it just as if it were straight.



This also is ideal. Strings are always more or less *stiff*.

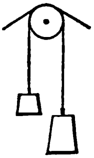
In working out problems of this kind we also generally suppose that the strings, rods, &c. do not alter their lengths during the action we are considering—which is the same thing as supposing the forces that stretch or compress them remain constant; or, at least, we suppose (what is usually sufficient for simplicity of treatment) that they alter so slightly during the action as not sensibly to modify the directions of the motions of the system, and *return* exactly to their first state at the conclusion of the action. It is usual also (though this is of minor importance) to neglect the mass and weight of mere strings, in comparison with the masses they connect.

On these hypotheses the general treatment of the problems of connected masses, so far as regards our particular subject, is very simple. For it follows from our definitions that, with connections such as we suppose—

The mutual reactions of surfaces, strings, or other connections, however much they may alter the motion of each particular mass in the system, both as to direction and velocity, yet when the sum of the work done by them (treated as forces, as above explained) is taken throughout the system, it must necessarily come to nothing.

In the case of motion over a surface, itself supposed immovable, this is so because the action of the surface is at every point at right angles to the actual motion (p. 40).

In the case of every other connection—such as two masses both in motion, sliding over and pressing on each other; or a string stretched between two masses tending to fly asunder; or a rod preventing them from coming closer together—as much work is done by the force on one mass as the other mass does against the force. Take, for instance, the case of a string passing over a pulley and attached to two weights. The string is pulling *against* each equally, by the Third Law. If, then, one mass is moving away from this force, it is *doing work* against it measured by the strength of the pull and distance moved over estimated in its direction. But, as we suppose the stretch of the string unaltered (or, if altered by a variation in the force during the action, yet restored at the end), the other end of the string moves just as much in the direction of its length, and *does* exactly the same *work* on the other mass; so that the sum of the positive and negative work (p. 37) is nothing. The same reasoning applies in the case of a pushing or pulling rod, and of two masses simply pressing against one another by their surfaces.

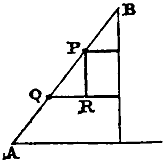


The consequence of this is that, taking the system throughout, the sum of all the changes of Energy of Motion, positive and negative, in passing from one position to another—which we have shown to be ascertained by summing up all the positive and negative work done on all the masses—is measured by the amount of work done by and against the 'impressed,' or 'external' forces only—the forces which would remain in action, if all these connections were suddenly severed.

A few examples will illustrate this Proposition.

1. Suppose a mass (m) to move down an inclined plane A B, from P to Q. Let v_1 be its velocity when at P, and v_2 at Q. Draw P R and Q R vertical and horizontal.

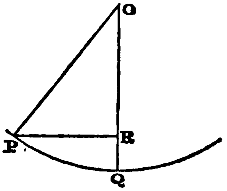
Then the work *done by gravity* is $m\ g \times$ distance fallen through, or $m\ g \times$ P R. Therefore $\frac{v_2^2}{2} - \frac{v_1^2}{2} = g \times$ P R.



The student who is unfamiliar with the subject may verify this by looking at it in another light. Let the inclination of A B to the horizon = A. Then the force of gravity ($g\,m$), acting vertically, may be treated as resolved into two forces, $m\ g$ sin. A, and mg cos. A, the first in the direction of the inclined plane, the other perpendicular to it. This latter is counteracted by the reaction of the plane, and the motion is produced, or altered, only by the force mg sin. A. acting from P to Q. The work done *by this force* is mg sin. A × PQ. But $\text{PQ} = \frac{\text{PR}}{\sin. \text{A}}$; whence work done is $mg \times \text{PR}$, as before.

A study of these two methods of arriving at the same result may serve to show that the work done may be indifferently measured, either, as heretofore, by the product of the whole force and the distance estimated in its direction, or by resolving the force into two, one at right angles to the actual motion, and the other in the direction of the motion, and taking the product of this latter component and the actual distance. When the body is moving freely, not along a surface, this latter method can only be pursued or fully proved by the help of the higher mathematics.

2. So of the pendulum (bob and string) which we took as an illustration of Energy. Here we have the motion directed by the pull of the string, but it does no work: the force of gravity is the only external force.



Let, then, V be the velocity when the bob is at its lowest point Q, and v its velocity when at any point P. Draw PR horizontal. Then, throughout the motion, upwards and downwards—

$$\frac{V^2}{2} - \frac{v^2}{2} = g \times \text{QR}.$$

If H be the greatest height to which the bob rises, $v = o$ when this is reached: therefore $\frac{V^2}{2} = g\,\text{H}$, or $\text{H} = \frac{V^2}{2g}$

3. Take, again, the case of our type of constant force (p. 15), a *weight** (P g) drawing a *mass* (W), by means of

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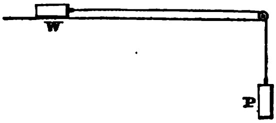
* That is, a mass P acted on by gravity. This is the proper meaning of weight.

a string passing over a pulley along a smooth table. Here P and W have necessarily the same velocity, and the distance moved over by each mass is the same as the *fall* of P.



If v_1 and v_2 be this velocity at two epochs, and if P have fallen through a distance (h), our formula gives us—

$$(P+W)\frac{v_2^2}{2}-(P+W)\ \frac{v_1^2}{2}=P\,gh^*$$

4. All the 'Mechanical Powers' described in Treatises on Mechanics, and all combinations of them which make up any machine, come under this principle, when friction, stiffness, and the like are neglected. Where the 'Power' and the 'Weight' are so adjusted with reference to the construction of the machine that, when the machine is set in motion by any means, the work done by gravity on the one, and on any parts of the machine which descend, is equal to the work done against gravity by the other and the parts which ascend, there will be no production or destruction of energy of motion; when no such balance is

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* The force stretching the string, and acting on W, can easily be determined. For, as the velocities of P and W are always the same, the forces acting on them and changing their velocities must be proportional to their masses (p. 16).

Let the force of the string be R.

Then the only force on W is R; and the force on P is $P\,g - R$. We must therefore have

$$\frac{R}{W}=\frac{P\,g-R}{P};\ \text{whence } P=\frac{P\,W}{P+W}g.$$

If, as in our original example, $W=1$, and $R=1$, we have

$$1=\frac{P\,g}{P+1},\ \text{whence } P=\frac{1}{g-1}=\frac{1}{31}\ \text{nearly.}$$

Therefore, to keep up the action of our constant unit of force while the unit of mass moves through a distance h, we must have gravity doing work to the amount $P\,g\,h$, or $\frac{32}{31}h$.

established, there will be an interchange of work done and Energy of Motion, as in our simpler cases.

Note.—In our examples we have supposed the strings either to move freely or to pass round fixed surfaces. In practice these surfaces are pulleys which revolve with the string. And, in a complex machine, with wheels, levers, &c., we have rotatory combined with rectilinear motion. The higher mathematics teach how to estimate energy of motion in a body rotating round an axis, so as to compare it with rectilineal motion, such as we have been considering; and then our Proposition still holds good: the whole energy of motion in the system, rectilinear and rotatory, increases and diminishes by the work done by and against gravity.

Where the force working the machinery is not gravity, we can equally well estimate the work done and power expended, if we can ascertain the average force applied. Take, for instance, the ordinary working of a steam-engine. The force is here the pressure of the steam on the piston; the action is reciprocating, backwards and forwards, but the connections are such that the ultimate motion communicated is continuous. Suppose the *average* force of the steam is F on each square inch of the piston—that is (to recur to our type of force), that it is equivalent to F strings, at unit-stretch, applied to each inch, and that the area of the piston is A square inches, l the length of the stroke, n the number of strokes made in an hour—then $F\,A\,n\,l$ is the energy that has been applied during that hour, expressed on absolute scale, being the product of the force by the distance moved over. And this must come out in—

1st. Increase of Energy of Motion in all parts of the system;

2ndly. Work done in raising weights; and

3rdly. In practice, in overcoming friction everywhere, if we do not assume 'perfect smoothness,' &c.

IMPACT OF PERFECTLY ELASTIC BODIES.

Bodies may also, while moving, meet with or overtake one another. We will take the case of two symmetrical bodies—say two cubes moving in the direction perpendicular to two of their faces, or two spheres—moving on the same line, either in opposite directions, and so meeting one another, or one moving faster than the other in the same direction, and so overtaking it.

The simplest case for our consideration is that of two masses (m_1) and (m_2) meeting one another with equal *momenta*; that is, where, v_1 and v_2 being their respective velocities in opposite directions, we suppose that $m_1 v_1 = m_2 v_2$.

When the masses come in contact each resists the motion of, that is, exerts force on the other; and by the Third Law of Motion these forces, and the momenta they respectively produce in equal times, are equal and opposite. And therefore, the momenta being originally equal, both will be destroyed simultaneously, and the bodies come, simultaneously, *to rest.*

The forces which thus come into play, in fact, arise from a strain or distortion of the two masses, and may be represented (as in the case of the magnet and iron, p. 25) by conceiving a spring—or two buffers, as between two railway-carriages—interposed between them.

If, then, the spring, at the moment of rest, becomes *set*, or broken, there will be no rebound, and all energy of motion will have disappeared. If, on the contrary, the spring be *perfect*, it will now expand again, driving back each mass with the same force, at each stage, with which it resisted the compression; and so they will finally part each with the same momentum which it had before impact, only in the opposite direction; and so the energy of motion which had been momentarily annihilated when the spring was at its greatest compression, will have been completely restored when the spring is completely unbent. The forces concerned in the compression and re-expansion are varying forces; but otherwise the case as to each mass is exactly analogous to that of the motion of a mass thrown upwards, having its motion first completely destroyed by doing work against gravity, and then completely restored by falling down again through a distance equal to that by which it rose.

Bodies whose behaviour is of the first kind are called 'perfectly inelastic,' while those of the second kind are usually described as 'perfectly elastic.' Both classes are ideal, like perfectly smooth surfaces; but lead and putty are fair approximations to the first class, and glass and ivory to the other. In nature, we may say that the spring is always more or less weakened, and so the rebound somewhat less than the compression, and some motion lost. We here confine our attention to the case of perfect elasticity.

If now, while m_1 and m_2 are approaching one another, a common velocity (u) be impressed, in either direction, on both of them, the *relative* circumstances of the impact and rebound will obviously be unaltered, and therefore (by the Second Law) the relative actions of the forces will be unaffected.

Suppose this additional common velocity to be in the direction in which m_1 is moving, its total velocity will become $u+v_1$, and that of m_2 will be $u-v_2$.

Before impact, then, the total energy of motion will be—

$$\frac{m_1\,(u+v_1)^2}{2}+\frac{m_2\,(u-v_2)^2}{2}$$

And after impact the velocities will, as above explained, become $(u-v_1)$ and $(u+v_2)$, and therefore the total energy of motion will be $\frac{m_1(u-v_1)^2}{2}+\frac{m_2(u+v_2)^2}{2}$; and it is easy to see that, if the quantities in brackets be multiplied out in both these expressions, the only difference between them will be, that in one u is multiplied by $m_1v_1 - m_2v_2$, and the other by $m_2v_2 - m_1v_1$. But we have made $m_1v_1 = m_2v_2$; and so each of these multipliers $= o$; and therefore the energy is the same both before and after impact.

Now, with whatever velocities the masses m_1 and m_2 may be moving, we can always represent the motion as made up, in this way, of a *common* motion (u) in one direction and two opposite motions v_1 and v_2 such, that $m_1\ v_1 = m_2\ v_2$.

For, let V_1 and V_2 be the actual motions, which we will suppose to be both in the same direction, V_1 being greater than V_2, so that m_1 overtakes m_2. We have then only to satisfy three equations to determine the three unknown quantities u_1 v_1 and v_2, namely,

$$\left.\begin{aligned} u + v_1 &= V_1 \\ u - v_2 &= V_2 \\ m_1\ v_1 &= m_2\ v_2 \end{aligned}\right\}$$

which are easily solved.

Therefore, the impact of perfectly elastic bodies makes no change in the total energy of motion when the rebound is completed. But when the elasticity is imperfect, the rebound is less than this, and there is a loss.*

If $V_2 = o$, or the body impinged upon be at rest, then it will be seen (either from the below-written formulas, or by a direct resolution of the three Equations, writing

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* If the equations are solved, we find

$$u = \frac{m_1 V_1 + m_2 V_2}{m_1 + m_2},\ v_1 = \frac{m_2(V_1 - V_2)}{m_1 + m_2},\ v_2 = \frac{m_1(V_1 - V_2)}{m_1 + m_2}$$

And the velocities after impact, being $(u - v_1)$ and $(u + v_2)$, will be found to be

$$\frac{2m_2 V_2 + (m_1 - m_2) V_1}{m_1 + m_2} \text{ and } \frac{2m_1 V_1 - (m_1 - m_2) V_2}{m_1 + m_2}.$$

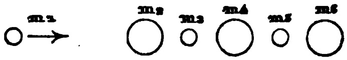
the second as $u = v_2$) that if $m_1 = m_2$, the impinging ball will come to rest, and the other go on with the motion the first had originally: if m_1 be less than m_2, m_1 will rebound with less velocity than it had, imparting some motion to m_2; if greater than m_2, it will send m_2 on, following more slowly.

If the impinging bodies, besides repelling each other during contact, attract each other when severed—as in our example of the magnet and iron, with an intervening spring (p. 25), or the action of a racket-ball and an elastic floor, with the force of gravity bringing them together—the energy of the rebound being equal to that with which they came together, they will separate until it has all disappeared in doing work against the attractive force: then this force will begin to do work on them, and finally will bring them together with the same energy which they had at the first contact. And so there will be an oscillatory motion backwards and forwards, which, if the elasticity were perfect and there were no air to carry off any of the motion, would continue for ever.

If we take as the definition of perfect elasticity 'the conservation of the full original energy of motion before and after the contact is over,' the two foregoing observations may enable the student to conceive that this quality may exist in two substances although the rebound is not complete, provided we take account of vibratory or tremulous motion in the parts of the bodies themselves, such as is sometimes conspicuous in a sounding-board, and which, at any rate, makes itself sensible in the musical tones it impresses on us through the air.

Suppose, first, a row of elastic balls (in our first sense) of various (say, alternately large and small sizes), at a sensible distance apart, and all at rest.

Let a small ball, m_1, impinge on m_2. It will, as we have seen, rebound more slowly than it approached, sending m_2 on towards m_3; m_2 will not rebound, but send m_3 on and itself follow more slowly; m_3 will rebound, after



sending on m_4, and so will meet m_2 again, and these two will part their energy between them again in some proportion dependent on the original circumstances, and so on; if m_6 be the last of the series, it will go on freely.

The whole energy of motion will remain at its original amount at all times at which there is no actual contact and pressure between the balls. But inasmuch as there is motion backwards and forwards between m_2, m_3, m_4, &c., the onward motion of m_2, m_3, m_4, &c., considered as a *whole* and *estimated in one direction*, will not be what it would have been had they all, when m_1 impinged upon them, been joined together in one perfectly elastic body, acting as we have supposed any two of them to act.

If now we suppose the masses to be set very close together, so as to appear to be one body capable of a slight vibratory motion *in its parts*, alternately compressing and expanding its length; and if we further suppose that there is an attractive force (what we call cohesion), which, allowing a *play* between them, acts to prevent them flying apart—adding, in short, another oscillating motion to them, as of the magnet and iron—we have a rough notion of the real action of two bodies of definite size impinging on one another, even if all their small parts are supposed 'perfectly elastic.' Going back to our first illustration of elastic action, it is not properly represented by a single spring between the two visible masses, but rather by supposing the whole mass made up of springs, each taking up the strain from the other, and requiring time within which to do so and to return the motion.

VARYING FORCES.

So far as we know, or can reasonably conjecture, all the primary forces in nature vary in magnitude and direction according to the relative position of the body acted upon. When we want to exhibit the action of a force really constant within the limits of our experiment we have (as in the case of the mass on the table) to *do some work* in order to maintain its constancy while the position is changing (see *note*, p. 50).

Thus, gravity, which we familiarly treat as constant in magnitude and direction, is neither one nor the other. It is only approximately so while we confine our observations to distances and spaces very small in comparison with the dimensions of the globe we live on.

Such forces, again, as the elasticity we have just been considering, or cohesion, &c., are only sensible when parts of the bodies which exert them are so close that, to our ordinary senses, they appear to touch; and we have reason to believe that, within those distances at which they are sensible, their magnitude changes at different distances.

As gravity is the most universal force in nature—absolutely universal, as far as we know—and as we know its laws of action thoroughly; and as a true conception of it, besides its intrinsic importance, will tend much to clear up difficulties which might otherwise beset the conception of 'Energy of Position, or Potential Energy,' to which we are coming, a somewhat full statement of its law, and of the results as regards energy and work done, may not be out of place here. If the formulas seem abstruse, the substantial results, which will be found stated at the conclusion of the section, will, it is hoped, be intelligible and sufficient for our purpose.

The force exerted on every particular mass is in truth the resultant of forces exerted on it by every other mass existing anywhere in the universe. And each of these forces is subject to the Third Law; that is, one particular mass exerts equal and opposite forces on each of those other masses. For simplicity (as our exposition is an elementary one) we will suppose all masses spherical, or made up of spherical parts.

Then, if m_1 and m_2 be any two masses, and r the distance between them, Newton proved that each exerts on the other an attractive force directly proportional to both of the masses (that is, to $m_1 m_2$), and inversely proportional to the square of the distance between them.* In other words, if, for simplicity and for our present purpose, we call the force between two units of mass at the unit of distance (1), the force between m_1 and m_2 at distance r will be $\frac{m_1 m_2}{r^2}$.

(a) Let us first suppose that one of the masses (m_2) is so large in comparison with the other (as the earth is with regard to a stone), that m_1 gives it no sensible motion. (See p. 25.) Then mathematical investigation, following the processes indicated p. 44 shows that, when m_1 is moving directly in the line of action of the forces (which is the case of the stone falling or rising vertically), the *average force* exerted while it passes from a distance (r) to another (r') is the *geometric mean* between the forces at r and r'—that is, $\frac{m_1 m_2}{r r'}$; and therefore the work done is $\frac{m_1 m_2 (r-r')}{rr'}$, if *by* the force, or $\frac{m_1 m_2 (r'-r)}{rr'}$, if *against* the force; that is, according as m_1 approaches or recedes from m_2. And therefore the energy of motion $\frac{m_1 v^2}{2}$ is increased or diminished by this amount.

If m_1 be not moving directly towards m_2, it is found—and the student who has followed the proof in the case of oblique motion with constant force will perhaps be prepared for it—that the *work done* is independent of the transverse motion, and is still $\frac{m_1 m_2 (r-r')}{r r'}$, or $\frac{m_1 m_2 (r'-r)}{r r'}$, as before.

(b) If we make no supposition as to the relative magnitudes of m_1 and m_2, and suppose them to move, by any path, from the position $m_1 m_2$ to $m'_1 m'_2$, changing their relative distance from r to r', then we find that the *sum* of the work done by m_1 on m_2 and by m_2 on m_1 is now represented by $\frac{m_1 m_2}{r r'}$ multiplied by the difference of the distances r and r'; and consequently, that the *total* energy of motion $\frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2}$ is increased or diminished by this amount, as the case may be.

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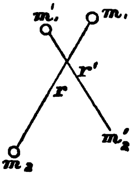
* This is the same as the law of the diminution of the intensity of light or heat, or other measurable quantity, when diffused from a centre. The amount of light on an inch of surface exposed perpendicularly to it will vary inversely as the square of its distance from the luminous body. But we do not know that there is any connection between the facts.



(c) And, finally, if we take any number of masses, m_1, m_2, m_3, &c., and mark the distances between them as $r_{1,2}, r_{1,3}, r_{2,3}$, &c., then (somewhat in analogy to the case of *connected* bodies), though the energy of each single mass may depend on the path it has followed, yet the sum of all the energies $\frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} + \frac{m_3 v_3^2}{2}$ + &c. is shown to increase and diminish by the sum of the work done, treating it as *positive* or *negative* as the case may be, between all the masses—that is, by

$$\frac{m_1 m_2 (r_{1,2} - r'_{1,2})}{r_{1,2}, r_{1,2}} + \frac{m_1 m_3 (r_{1,3} - r'_{1,3})}{r_{1,3}\, r'_{1,3}} + \frac{m_2 m_3 (r_{2,3} - r'_{2,3})}{r_{2,3}, r'_{2,3}}$$

+ &c.

The important point to note in this investigation is, that the total work done, and the changes due to it in the total Energy of Motion, depend entirely on the changes in the *relative distances*, with no regard to the paths by which the masses have moved; and that, though we should be quite wrong if we reckoned the change of energy in any one mass as determined by the work apparently done (according to our formula) between it and each of the other masses, yet whatever error we should fall into as regards each one mass would be made up in the others; and in the whole system, the change of Energy of Motion is calculated from the work done between each pair of masses as if they had been alone in action, and had mutually and directly approached and receded, as much as in fact they have done.

And these two results are not the consequence of the particular *law*—of the inverse square of the distance—

which gravitation follows, but solely of the other fact, that *the forces between each pair of spheres is directed in the line joining their centres, and has everywhere the same magnitude when their distance is the same, whether the masses be in motion or at rest.*

If, then, we knew that the forces that come into play when bodies are in close contact (such as cohesion, elasticity, &c.) are of this kind, we should conclude that the work done between the particles making up the bodies is in this way dependent on the changes of distance between the particles (that is, generally speaking, on the amount of compression, expansion, and change of shape of the bodies); and, on the other hand, if we find experimentally that the results as to the final motion of the bodies and their changes of shape, &c. seem to be such as they should be on the above supposition as to the nature of the forces, it is an argument, as far as it goes, for the truth of the supposition.

If, for instance, glass, or any other substance, had been found to be perfectly elastic, instead of only approximately so, we should with very great plausibility have inferred that the force of repulsion which comes into play when two balls of such material come together, depends, at each stage of the action, only on the amount of compression, and not on the direction of the velocity *to* or *from* the centres; because the energy of motion destroyed in the approach would be (in such a case) always exactly restored in the rebound.

Note.—We have hitherto used the intelligible phrase 'Energy of Motion' for the quantity $\frac{m\,v^2}{2}$ and sums of such quantities. It is in frequent use. But a more concise name, derived from the Greek, and with the same meaning, is 'Kinetic Energy.' We will use it for the future whenever convenient.

II. ENERGY OF POSITION, OR POTENTIAL ENERGY.

If now we review the conclusions we have either reasoned out or stated as the result of more abstruse investigations founded on the principles we have established, we find:—

(1) That when a mass (m) is moving with velocity (v) it has, in virtue of it, a Dynamical Energy numerically measured by $\frac{m\,v^2}{2}$ on the absolute scale, that being the measure of the work it can do against a force before it is entirely exhausted; and it is diminished in exact proportion to the work done up to the point of exhaustion. If it encounters no force, it retains this energy by the First Law.

(2) That when the force acts with the motion, instead of against it, it adds kinetic energy in exactly the same measure.

(3) That force is effective in consuming or increasing kinetic energy only so far as it acts in the line of the actual motion; its component at right angles to this direction only acts to change the direction.

(4) That whenever the force acting on a mass may be treated as sensibly directed to a fixed centre (see p. 58, a), and as depending only on the distance for its magnitude, or when it is a constant force always acting in the same direction; the work done by or against the force while the mass moves from one known position to another is determined solely by the knowledge of *those first and last positions*, and of the way in which the force depends on the distance—the *Law of Force*, as it is called—and not at all upon the actual path described.

(5) That when the forces acting on a number of masses—a given 'system,' as we call it—are forces exerted between the masses, and dependent solely on the distances between them; and where all the connections fulfil the

ideal conditions indicated by the phrases 'smooth surfaces,' 'flexible strings,' 'perfect elasticity,' or the like, which, after acting, *leave no work behind them*, positive or negative; then the whole sum of work done in the system, as its masses pass from one set of positions to another, and the consequent change in the sum of all the kinetic energies, will be determined solely by the relative changes of position of the masses, and the initial and final distances between them, just as if the pairs of masses had approached or receded each from the other independently, as in (4), or rather, as in p. 58 (b); although the kinetic energy of each individual mass or pair will be different according to the paths pursued; the excess gained by some being made up by loss in others.

Now, the proposition we began with announcing is, that when any energy is gained or lost in one mass or set of masses, an equivalent is lost or gained elsewhere. We ought, therefore, to be able to show that, whenever work is done against a force, and so kinetic energy lost, an exactly equal power to do work of some kind—to produce measurable change—is gained somewhere or other, and *vice versâ*.

And there is no difficulty in pointing it out when the system is of the character indicated in (4) and (5).

Where the effective forces (those which do work) depend on the distances between the masses alone, the amount that can possibly be done between each pair of masses is a definitely limited quantity, made up of the work that can be done between the parts into which that distance may be divided.

Thus, if m_1 and m_2 act on each other by gravitation, and pass from distance r to a lesser distance r', the work done by the force is, we have seen, $\frac{m_1 m_2 (r - r')}{r\, r'}$ or $m_1\, m_2 \left\{\frac{1}{r'} - \frac{1}{r}\right\}$. This work is done in the shape of added kinetic energy if uncounteracted, or of work against the forces if others are in the field,—and the power to do it is *spent*; only to be restored as a power by the expenditure of some work *against* the mutual attraction, bringing the distance back from r' to r. And if we keep to our assumption of the attracting bodies being spheres, or made up of spherical parts, the whole work that can be done is what would be done by allowing the two masses to come together, separated only by their own bulk—by the sum of their radiuses; at which instant the forces that determine the ultimate cohesion, consistence, and resistance of matter come into play. And so of other attractive forces.

If there be repulsive forces between masses in a system of this kind, a theoretical difficulty might occur to the student, inasmuch as there is no such obvious limit to the power of doing work between them, because that work would consist in driving them apart, and they may become more and more apart indefinitely. But this need not trouble him; for in nature all forces are found to diminish in intensity as the distances of the acting masses increase, at such a rate that there is a definite limit to the work that can be done between any one position and the extremest distance we may choose to contemplate.

The really permanent dynamical forces other than gravity known to us—those that cause elasticity, cohesion, &c.—diminish so rapidly that they either become non-existent or so immeasurably small as to elude all observation and experiment, as soon as there ceases to be *apparent contact* between the surfaces of the acting bodies; so that no work is done while the bodies move anyhow outside this distance.

Magnetism and electricity, which cause repulsion under certain circumstances, may seem an exception; but, besides other answers that might be given—even treating them as permanent forces, like gravity—the work they can do may be shown to be limited, as above stated.

For these forces follow the law of gravity as to the variation of their magnitude—diminishing as the inverse square of the distance. So that the work done by the force while the electrified masses pass from a distance r to a *greater* distance r' is $F\left\{\frac{1}{r}-\frac{1}{r'}\right\}$, where F is the force at distance (1). This measure of the work done increases, it is true, for ever, as r' is increased, or as $\frac{1}{r'}$ diminishes, because the smaller the quality to be subtracted the greater the remainder; but that remainder can never quite reach $\frac{F}{r}$, which is therefore the *limit* of the work that can be done.

With regard, then, to all masses acting on each other, whether by universal gravitation or by any other forces having the character above described, there is, in virtue of the relative distances at which they may at any time be apart from each other, a capacity in the system for doing positive work (work *by* the mutual forces); limited in amount, because the very utmost that can be done without being again undone, is the sum of all that would be done if they were made to move (whether by their own kinetic energies or mutual action, or by any impressed, external forces applied to them) to the relative positions in which they could none of them do any more sensible work—the attracting bodies in contact, the repelling ones beyond the sphere of their sensible action.

This limited amount of work is the measure of what we may call the 'Absolute Energy' which the system has in virtue of the position of its masses—or, more shortly, its 'Energy of Position.'

Another name, shorter still, and more often used, is 'Potential Energy'—meaning power of giving itself kinetic energy (which latter kind of energy is sometimes called 'Actual Energy').

And our reasoning shows that this Potential Energy increases and decreases by the same measure and quantity by which the kinetic energy decreases and increases, so long as no force external to the particular system alters either energy. In other words: *The sum total of the*

Kinetic and Potential Energies of any system thus constituted remains constant during all the changes of position that it can give itself. This is what is called the principle of the 'Conservation of Energy' as regards the dynamical changes of a system of this kind.

It is obvious that this absolute Potential Energy cannot be discovered and expressed numerically, if we take into account, as forming one system, all the bodies that act, or can in some position act, on a given mass.

For instance, there is potential energy between a stone and the remotest fixed star: for if the stone were impelled with such force from here as to get beyond the preponderating influence of the earth and of the sun, and so proceed till it fell on to the star, the whole work done between stone and star would, by the formula already given, be nearly $\frac{\text{mass of star} \times \text{mass of stone}}{\text{radius of star}}$. So, again, there is energy between two globules of quicksilver, while they are lying on the table; for these bodies, if brought to touch, will run together, do work on each other, and so exhaust their power.

But, except for the purpose of enunciating in the clearest language the general principle which is in fact proved by the reasoning of the last section, we do not need, in dealing with any particular case, to trouble ourselves with absolute energy, but only with the changes in it which take place in the course of the motion—just as for ordinary questions about heat we do not want to know the whole quantity of heat in a body, but only how much is gained or lost when its temperature changes from one arbitrary line on the thermometrical scale to another.

Just, then, as with the thermometer we fix an arbitrary *zero-point*—and that not necessarily the same in all thermometers—so, in each problem, we may, if convenient, take a zero-point (so to say) of Position and Energy of Position, from which to reckon; and, if we please, to reckon upwards and downwards, treating the quantities below this as *negative.*

We may, in the first place, leave out of account and treat as non-existent all potential energy which we know does no work, or none that is of a magnitude worth taking account of, in the course of the action we are considering.

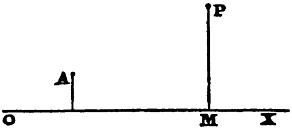
Thus, when we are treating of 'Projectiles'—cannon-balls, and the like—we may confine our attention to the force of gravity, as ordinarily estimated in the latitude &c., we are dealing with, although the sun, and moon, and neighbouring hills, are all exerting force; because calculation shows that those forces are so small that the work they can do in the course of the longest flight a cannon-ball can take is as nothing compared with the foot-pound work of the earth's attraction. The moon, for instance, is sixty times as far off from the projectile as the centre of the earth is, and its mass is about $\frac{1}{80}$th of that of the earth. The force, therefore, by our formula, is $\frac{1}{80 \times (60)^2}$ or $\frac{1}{288,000}$ of the earth's, and the projectile and moon must approach or recede by 288,000 feet, or over 50 miles, to do a single foot-pound of work between them.

Again, although, in strictness, the work is done *between* each pair of masses, yet when one is enormously greater than the other we may deal with the case as above explained (p. 58, a), and speak of the energy as belonging to the smaller body exclusively, and measure its changes by the changes of position of that body relatively to the larger one.

We will now show by examples how this way of looking at the facts enables us to treat the ordinary problems connected with projectiles and elementary machines in a convenient way.

(1) The motion of a body projected into the air comes to an end when it reaches the ground. We have then only to consider the changes of its energies of motion and position from the point of projection to the level at which it may touch the ground.

We may therefore take as our *zero* or starting-point of Energy of Position a level plane, the lowest at which our projectile can, under the circumstances, reach the ground. Let, then, O X be such a plane—the floor of a room, the ground-level in an open plain, the sea-level, &c., as the case may suggest. Let P be the position of the projectile (of mass m) at any time, and let the height P M above the level be h. Then, taking (on this principle) the energy of position on the level O X to be zero, it will be $m g h$ at P—that being the work that would have to be done against the force to raise it to that height.



If, then, v be the velocity at P, the sum of the two energies (kinetic and potential) is $\frac{m v^2}{2} + m g h$.

and this must remain unchanged throughout the motion, as long as no other forces than gravity sensibly affect it.

Suppose then that the point A, at the height H above O X, was the point whence the mass was projected with velocity (V), or that by any means we know the velocity at this particular point of the actual path. Then $\frac{m\ V^2}{2} + m g \mathrm{H}$ was the sum of the energies at A, and we must have throughout the motion

$$\frac{m\ v^2}{2} + m g h = \frac{m\ V}{2} + m g \mathrm{H}, \text{ or } \frac{v^2}{2} + g h = \frac{V^2}{2} + g \mathrm{H}$$

This equation enables us to determine the velocity at any height, or the height corresponding to any velocity during the upward and downward motion, without calculating the actual path followed.

(2) In the case of the pendulum, the level below which the bob cannot fall is that which it reaches when it hangs vertically, and we may measure the potential energy from that point. The formula we get is that of page 49.

Note.—If the string be sensibly extensible it will be unequally stretched at different points in the motion, and we should have to take account of the work done, or the changes in the potential energy of the string, which would make the problem very complicated; too much so for an elementary treatment.

(3) In the case of any complicated machinery, of weights, wheels, pulleys, levers, &c., set in motion and left to the sole action of gravity, we may take as our zero-level the floor on which the machine rests, or a level below which none of the weights descend; then the energy of position of the whole system may be taken as the sum of the products of each weight which *alters its level* during the motion multiplied by its height *above* the floor (if we choose this level), *diminished by* the like products of any weights that may hang below the floor (as when weights are being raised out of a pit) by their distances below that level. (See pp. 65, 66.)

The energy of motion will have to be expressed, as in p. 51, so as to take account of rotatory motion of wheels, &c.

And then, if we know the positions of all the weights at the moment the machinery is put into gear, when there is no kinetic energy, we shall have, throughout the motion, the sum of the kinetic and potential energy in any given position equal to the original potential energy at starting.

As, in most machinery, the motions of all parts are known when that of one is—there being generally no independent play of the parts allowed—this Equation tells us the whole course of the motion and work done until some weight touches the ground.

We have seen, so far, where that energy is to be found which interchanges with kinetic energy, when the forces which do work are *permanent*—always ready to act in the same way when masses are in the same relative positions, and dependent for their magnitude and direction solely *on those positions*—not on the velocity or other condition of the masses; and we have given it a name.

But we have already given examples of forces which do not—at least, apparently, and at first sight—come under this description. The friction of the furrow, for instance (p. 33), only acts to oppose motion—it never originates it: and it changes from one direction to another as the motion does. The forces causing a rebound in the bodies which are imperfectly elastic, or unelastic, in the stricter sense—that is, even taking account of the internal vibrations (see p. 56)—are another example: they oppose the compressing energy more than they urge on the rebound.

We now see that the work done against these forces always exhausts an existing energy: kinetic, when a mass in motion is gradually or suddenly brought to rest by them; potential, when it is dragged along a surface by the force of a weight over a pulley or the like appliance of gravity; or of that kind which resides in the animal frame (p. 32) when it is the work of horses. We must therefore seek for an equivalent gain elsewhere.

THERMAL AND OTHER ENERGIES.

IMPACT, FRICTION, AND HEAT.

We have, then, so far as the limits of an elementary treatment permit, gone through our illustrations and proofs of the Doctrine of the Conservation of Energy as it was known and expounded, under a different name, up to half a century ago, and as it would apply to natural actions of a dynamical character, if all motions were either of masses moving in empty space with no contact, or of masses connected by perfectly flexible strings, or sliding along perfectly smooth surfaces; and if impinging bodies were all perfectly elastic.

In such a world, however varied the play of motions might be, the sum total of the kinetic and potential energies would remain the same. The planetary motions exhibit a sensibly perfect specimen of such a system. There is, indeed, reason to suspect that the space in which these motions take place is not absolutely empty—that it is pervaded by extremely attenuated matter. If so, collision must be constantly taking place, and, we must suppose, friction also; but no signs of this have been detected in the motions of the large masses—the planets. Accordingly, from age to age, their velocities and their relative positions, are connected by this Law of Conservation. If we know what the exact positions of all the planets and satellites are now, and what they were a century ago, we can tell what the difference is between the total kinetic energies.

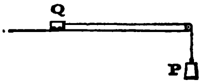
But these conditions do not exist on earth. Whenever dynamical work is done, the dynamical energy—kinetic and potential together—which is left to show for it is always less than what was spent, and the most prevailing causes of this apparent failure are collisions and friction.

There are other causes, electric phenomena, &c., and there are cases where we may fancy dynamical energy has been destroyed, when it has in reality only become invisible. For instance, the resistance of air to the motion of a mass, though it may be partly in the nature of friction and collision, is in great measure the result of motion given by the moving mass to the air by pressure—a true interchange of ordinary kinetic energy given up by the solid mass with kinetic energy imparted to the mass of the air. So, again, the vibratory motion which may be left after collision in perfectly elastic bodies (p. 56), or when we apply the force of our hands or breath to play a musical instrument, partly passes off in the form of *wind*, however gentle, and partly in that other more subtle vibratory motion of the air which, conveyed to the ear, produces the sensation of sound.

But, after making all these allowances, there remains a farther loss, very commonly far the largest part of the whole, which cannot be traced to any resulting *dynamical* effect, as defined at the beginning of this treatise, or usually so called in books written before the most recent times. The only effect discoverable is most often, as mentioned above, *Heat.*

That heat is produced by impact and friction can never have escaped observation. But the discovery that the *quantity* generated is always exactly proportionate to the quantity of dynamical energy apparently lost, is a recent discovery, and is perhaps the most important contribution to our knowledge of the laws that connect all parts of nature that has been made since the time of Newton.

The example of work done with no resulting Potential Energy given p. 33 is a rough one. The plough does leave some other traces besides heat—raised clods, cohesion overcome in the moved soil, and increased, perhaps, in the subsoil. We may take more typical cases, where no other effect, or none worth considering, is left. Take, for instance, the case of the horizontal and vertical masses already given, only with the table not 'perfectly smooth.' The weight $\mathrm{P}g$ will drag along the mass Q, but the kinetic energy of the two will always be less than what is due to the potential energy lost, $\mathrm{P}gh$.

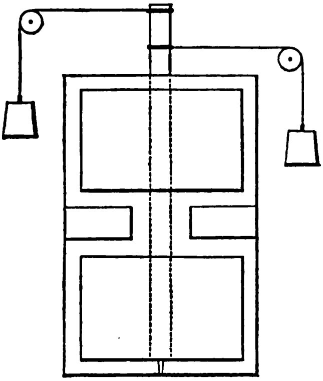


Now, the proportion of P to Q may be so adjusted, with reference to the friction of Q on the table and the string on the pulley—and in strictness we should add the stiffness of the bending string, (p. 47)—that the case may be like that of our type of work done (pp. 32 and 36); the motion may be started by the hand, and then the force of gravity on P may be just sufficient to balance these resisting forces, and keep the apparatus going at the uniform rate so impressed upon it. And if the materials of which Q and the table are made and the degree of polish be judiciously chosen, there may be no perceptible change in these qualities after hundreds of repetitions of the experiment. In this case, then, if the supposed equivalence between the heat gained and the dynamical energy lost really exists, we ought to find, very approximately, a definite proportion between the heat evolved and $\mathrm{P}gh$; and the proportion thus found in one experiment ought to appear also in any other similar one.

A case where the materials are still less liable to be in any degree changed in character is where the friction, or collision, is between a fluid and a hard metal. If water be shaken in a tin pail and then let alone, it will soon come to rest. Its kinetic energy will have disappeared, and, if evaporation be prevented, neither water nor pail will show any change whatever, except in temperature.

It was by experiments of this kind that Mr. Joule fully established the equivalence in question.

He constructed a machine like a churn, complicating the interior with compartments, so as to check as much as possible any motion given to the fluids. The churn was worked, humming-top-wise, with two equal weights hanging over pulleys, pulling at the opposite ends of a string wound round the axis of the churning vanes. A scale of feet and inches, placed vertically by the side of the weights, would show through what distance they have moved from time to time; and if the proportions of everything are so arranged that *uniform motion* is attained before the close of the experiment, this final velocity can be measured also by the scale. The experiment being then stopped by disconnecting the weights, the fluid would very soon come to *complete rest.*



Suppose the sum of the two hanging masses to be w, and the distance fallen by each of them h. We should then have spent potential energy to the amount of w $g\,h$. And the only dynamical energy we should have to show for it finally would be the kinetic energy with which they left off, $\frac{\text{w}}{2}$ × square of final velocity. The quantity of heat produced in the fluid and the other parts of the apparatus ought, therefore, to account for the difference between these two quantities.

The heat evolved about the pulleys and string could not well, by direct means, be ascertained and separated from the rest. But the amount of potential energy which has actually been consumed in this part of the apparatus can be detected and allowed for on the principle explained in the last illustration.

We can disconnect the upper part of the axis from the vessel, adjust the string so that the weights shall pull *against* each other, and then try what *excess of weight* must be applied on one side, to overcome friction, and give the same velocity that was obtained in the full experiment. Let this be $\mathrm{P}\,g$. Then, out of the whole potential energy $\mathrm{W}\,g\,h$ spent in the experiment, an amount $\mathrm{P}\,g\,h$ has gone to overcome the friction in the pulleys, &c., and $(\mathrm{W} - \mathrm{P})\,g\,h$ is what has eventually produced the final kinetic energy and the heat.

If then the vessel be worked in a calorimeter (such as that described p. 30, or any other similar contrivance) we can ascertain how much this heat amounts to, and observe the proportion between thermal gain and dynamical loss.

Similar experiments were made on various substances—water, quicksilver, alcohol, oil, &c., for the liquid; brass, iron, wood, &c., for the churning vanes. And the more carefully the experiments were conducted the more did the results agree; and the conclusion now received is, that *the expenditure of* 772 *foot-pounds of dynamical energy, when all spent on the production of heat, by whatever intermediate contrivances, generates one unit of heat.*

A great number of experiments of the most various kinds, and by different persons, have also been made—not all of them concerned with friction or collision, but with these conditions running through them all: that energy was spent with a *dynamical* result less than the proper equivalent; and that the electric, chemical, and other physical characters of the bodies operated upon (however much they might vary during the experiment) were the same at the beginning and the end of the experiment. And the missing energy was always found replaced by heat, and always (allowing for the inevitable want of perfect accuracy in such delicate measurements) in the same proportion as above mentioned.

Hence we conclude that, in the interchange of physical actions, 'The unit of heat added or subtracted to or from any system is the *thermal equivalent* of 772 foot-pounds, or 772 *g* absolute units of dynamical energy, kinetic or potential.'

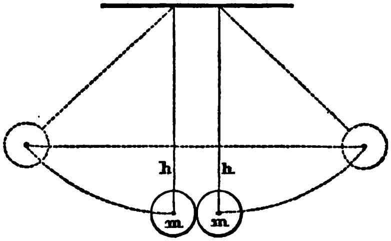
A few miscellaneous examples will familiarise the student with the meaning and application of this fact.

(a) Suppose 9 lbs. of iron to fall (in vacuo) through 772 feet, and then to plunge into a vessel containing 8 lbs. of water, both iron and water being at temperature 60° Fahr. The water will check the motion, and we may suppose the final clash on the bottom not to break up the mass of iron, or otherwise sensibly alter its quality; and after a time the disturbance of the water will cease and all come to rest, and, finally, to one uniform temperature. Supposing no sensible amount of heat to have escaped into the air or the coats of the vessel, the dynamical energy 9 × 772 foot-pounds has all passed into heat, and should therefore have generated 9 units. Now (p. 29) 9 lbs. of iron require one unit of heat to raise the whole from 60° to 61°. The other 8 units will serve to heat the 8 lbs. of water to the same temperature: and so the whole will stand at 61°.

(b) Suppose the experiment with Clarke's magneto-electric machine (described in page 7) concluded without using the wire to decompose water. The end will be simply the heating of the apparatus. If we collect and measure this heat—for example, by plunging the whole into a vessel of water—we should find the total quantity of heat generated equivalent to the kinetic energy which the same expenditure of steam in our steam-engine could have given to a fly-wheel, or to the foot-pound work it would have done in pumping or raising coal.

(c) Let two balls, of equal mass m, hang side by side in contact, as in the figure, by strings attached to a horizontal beam; let them be each drawn aside through the same angle, and be raised to the same height (h) above their original position; and then let go. Then—

The kinetic energy with which they clash will be $2\ m\ g\ h$, or $2\ m\ h$ in foot-pounds.



If they be perfectly inelastic all this will be lost as dynamical energy, and will be interchanged for $\frac{2\ m\ h}{772}$ units of heat, or 'thermal energy,' as we may call it for uniformity of expression.

If they be imperfectly elastic they will rebound, but with less velocity than that with which they came together, and each rise to a height h', less than h: $m\ h'$ will be the foot-pound energy of each, due to the kinetic energy with which each rebounded, and the loss will be $2\ m\ (h-h')$, represented by $\frac{2m\ (h-h')}{772}$ units of heat.

Note.—It is not always—perhaps, in strictness, it is never true—that this passage from visible motion to heat takes place *immediately*. Vibratory motion in the interior of the masses is, at all events, part of what takes the place of the visible rebound (p. 55). And unless the whole apparatus is confined in a close vessel, some of the vibratory motion will escape into the air, and only be finally replaced by heat *in consequence of friction in the air.*

If the student will now look back to the typical series of changes, each dependent on its predecessor, with which we commenced (p. 5), he will see that we have attained to *principles* on which they may be measured and compared, up to the point where 'change of physical condition' makes its appearance, in the conversion of water into steam: although the actual calculation, besides the practical difficulty in ascertaining the amount of friction, &c. would involve higher mathematical knowledge than he is supposed to possess. Thus, looking at the thing roughly, and leaving out of consideration the resistance of the surrounding air, we have—

1. The fall of the water from the level at which it issues from the reservoir to that where it encounters the water-wheel.

If it falls directly downwards, the kinetic energy at the lower level of each 1 lb. of water will be measured, in foot-pounds, by the height in feet fallen through; and if it runs down a sloping channel, the only difference will be due to the friction. If we *observe* the loss of motion, as compared with the calculation, we shall know what heat has been generated.

2. The wheel and the other mill-gear are so connected that we know the proportion of the velocity of every part to that of any one part, as of the outside of the wheel, where the water impinges. And then mathematicians can calculate (p. 51, *note*) what straightforward motion of the whole mass of wheel and gear would be equivalent, in its resistance to the impact of the water, to any given rotatory motion, and *vice versâ.*

We should then know, by the principles we have established in respect of impact, what the effect of the impinging water would be on the machinery, *if there were no friction* of the parts; and what the loss of kinetic energy and the equivalent of heat produced by the impact. The deficiency found in the actual result will measure the heat produced in the gear and in the water where the millstones are.

There will be two stages in the working. At first starting, the water impinging on the wheel while at rest will give it and the gear, &c. more and more motion, each portion of the water being more completely checked in its speed, and resting longer on the wheel, than in the second stage; in which, every motion being accelerated, the energy applied will be just sufficient to overcome the friction of all the parts, and the motion and production of heat will both be *steady*.

We now proceed to examine the phenomena of *boiling water* and cognate changes, though very briefly, and only so far as is necessary to get an adequate notion of the measure of the action compared with other work done.

CHANGE OF PHYSICAL CONDITION.

THE very oldest speculation on Physical Science that we know of is the doctrine of Thales of Miletus, about 600 B.C.: that *water* is the substance of all things. Perhaps, could we cross-examine him, his essential meaning might be found to be, that all things can, like water, exist in the fluid, the solid, and the aeriform, vaporous, or gaseous state.

At any rate this seems now a probable opinion. Almost all simple substances are known in at least two states; and those which, while simple, are only known as solids, as carbon, can form part of combinations which are gaseous.

These changes of physical condition depend on the application or withdrawal of heat; or of mechanical pressure.

Universally, if a solid can become liquid, or a liquid assume the vaporous or gaseous state, the application of *heat* in sufficient quantity will effect the transformations in this direction, and the withdrawal of heat will reverse them. And, with one or two exceptions (the freezing of water being one), extra *pressure* tends, in the opposite direction, to liquefy vapour and solidify liquids.

Referring the student to works on Heat or General Physics for a full treatment of the subject, we will here only say what seems necessary to exemplify our principle, and show that there is a numerical equivalence in these changes.

As explained in the section on the Measurement of Heat, a scale of temperatures has been established, experimentally, and we know how much heat is required to raise a pound of any substance from a temperature $t°$ to another $t'°$, while it undergoes no other change.*

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* To speak strictly, the experiments usually made on solids and liquids are so conducted as to allow some change besides that of temperature, for the substances are allowed to expand and contract freely under the pressure of the atmosphere. Thus, take the familiar instance of quicksilver, to which a certain quantity of heat is applied. It produces three separate effects: 1st, it increases the temperature of the substance; 2nd, it alters the physical condition of the substance, expanding it; 3rd, it does dynamical work: for the expansion is a motion of the surface against the force of the air which presses on all sides against it.

These circumstances have to be borne in mind, and allowed for as occasion may require; as when we are dealing with bodies under different pressures, or when comparing the case of free expansion under some pressure with one where the body is prevented from expanding at all, by continually increasing the pressure as more and more heat is applied. But in the case of most solids and liquids the difference is very small, and we shall only pursue this subject (in a subsequent section) in the case of vapours and gases, when the results are important.

But if we are experimenting on a liquid—say water—and do not prevent evaporation, our experiments will soon be vitiated. We may retain the vapour in the chamber, and take account of its mass and its temperature (which latter will be the same as that of the remaining liquid), but that temperature will be sensibly less than what the quantity of heat applied should produce according to the above-mentioned scale. In other words, *evaporation reckons as work done*, and heat disappears in proportion; just as dynamical energy disappears in the production of heat. And when 'boiling-point' (212° F. or 100° Cent.) is reached, no amount of heat will sensibly raise the temperature till all the water has been converted into steam.

We will here, by way of example, show how the numerical equivalence is made out in this case.

While the water is kept boiling let the steam be collected—as in a still—by covering the vessel with a hollow lid, whence a small thin pipe is led into a vessel of cold water, through which it passes in a long coil, sufficient to cool it down and condense it into water again; all proper precautions being taken to prevent the escape of heat and maintain the steam, as it reaches the coil, at the temperature of 212°.

Suppose the water in the cooling vessel to be at 60°, and its mass M. Let the experiment go on, keeping the water stirred, so as to acquire the temperature of the pipe, until it has reached to 61°. Then disconnect the pipe from the boiler, and ascertain the quantity of water in the coil: let this be m.

Then a mass m of steam at 212° has, first of all, passed into water at 212°, and then been cooled down to 61°; and this operation, when complete, has warmed a mass M of water by 1°, besides the metal of the coil.

The cooling of (m) from 212° to 61°, or through 151°, will, we know, yield 151 units of heat, very nearly (we might find the exact quantity in tables p. 30). And we also know how to allow for the quantity necessary to heat the coils, of known material and weight, from 60° to 61°; call this n units: then we have $151m-n$ units from this source gone to heat the water in the vessel. This would raise it $\frac{151m-n}{M}$ degrees. The excess of heat in the water, $M\left\{1-\frac{151m-n}{M}\right\}$ has been caused by the conversion of the steam (m) into water, without change of temperature.

When such experiments are made we find that the mass (m) of steam undergoing this conversion does as much 'thermal work'—heats as much cooler water—as $965m$ of water does in losing one degree of temperature.

If, on the other hand, we put ice at 32° (freezing-point) in a vessel of water at 61°, until the whole is reduced to water at 60°, similar calculation and reasoning show that it takes 144 units of heat, out of the water, to convert 1 lb. of ice at 32° into water without change of temperature.

Therefore, if there are such permanent numerical equivalences in nature as our theory requires, the conversion of 1 lb. of ice into water is equivalent, as work done, to the raising of 144 lbs. of water from 60° to 61°; or to raising 144×772 lbs. weight 1 foot against gravity; or to giving to 1 lb. of matter of any kind a velocity of $\sqrt{2 \times 144 \times 772 \times 32}$, or near 2,700 feet per second (by the equation $v^2 = 2gh$, or $v = \sqrt{2gh}$).

And the conversion of 1 lb. of water to steam at 212° is numerically expressed in the same way, substituting the number 965 for 144.

Many series of experiments of this kind have been made on every kind of substance which can subsist in two or more physical states, with perfectly consistent results, establishing the scale of equivalence between quantity of heat and change of physical state, where no other work is done or undone; both when the changes are, as above, from solid to fluid, requiring heat, and in the reverse order, requiring heat to be given out from the changing body to surrounding space; and we may take this branch of our doctrine as long ago settled.

It may, perhaps, be taken for granted, after what has been shown in this and the preceding section—and at all events experiment will prove—that when dynamical work is done (as in Joule's experiment), with no dynamical energy to represent it, and the substance is in a condition in which the direct application of heat would, wholly or partially, go to changing its physical condition; then that change will be effected to the precise amount required by the equivalence established between dynamical and thermal energy. For instance, if our millstone work in water which is already at the boiling-point, for every 772 foot-pounds of work done on them by the mill-stream and intermediate gear they will evaporate $\frac{772}{965}$ lbs. of water; while, at a lower temperature, they will partly heat and partly evaporate water, so as to make up the same total according to the scale established by experiment.

While men of science were unobservant of the fact of the production of heat from mere motion—while they did not speculate upon it, though they knew it in many cases—they were inclined to consider heat as a *substance*, a kind of matter; although everything was against the notion that it had that one property which is the essential dynamical characteristic of matter—subjection to the laws of motion and force. And, with this notion in their heads, they explained the above-mentioned facts by assimilating water to a chemical combination of ice and the substance of heat—caloric they called it—and steam to a combination with a larger quantity of it; just as, in chemistry, we have compounds of sulphur with one, two, or more proportions of oxygen. Then, just as the taste of the sulphur disappears in sulphuric acid, the warmth might disappear from the water. And so the phrase came into use of 'latent heat,' as expressing the quantity of heat that was *in* the steam, over and above what was in water of the same temperature. Now that we have distinctly perceived that, by merely stirring it, we can get *as much heat as we please* out of water, extracting it gradually at low temperatures, by placing cold bodies in contact with it, and leaving the water with exactly the same properties it had before, it seems impossible to conceive thus of heat. And if we are to seek for a phrase applicable to the facts, we might speak of steam as having potential heat, or potential thermal energy, in comparison with water, and water in comparison with ice, just as we speak of the raised weight having potential dynamical energy in comparison with one on the ground. But the phrase is not in use.

ENERGY DERIVED FROM HEAT.

We have seen that quantity of heat may stand as the representative of a dynamical energy which has been *spent* in producing it; and, so far, should rank as 'work done.' We know already that a heated body can expend some of this thermal energy in heating a colder body; and that this action will leave the total amount of energy unaffected. We will now farther enquire whether, just as kinetic and potential energy in Dynamics can *interchange*, so thermal energy can, to any extent, be employed to reproduce either one of them; that is, directly or indirectly, *do dynamical work*.

Now, as to direct action, we have already seen that it is almost the invariable rule—though there are important exceptions—that heat imparted to a body expands it against the pressure of the atmosphere, or whatever medium may surround it at the time, unless simultaneously an increased pressure is applied externally to prevent the expansion. The familiar means by which we measure temperature is an instance. Heat applied to the mercury, or other fluid in the thermometer, expands it; and if the tube be open to the air, the column rises against the atmospheric pressure which, as we see in the barometer, tends to press it down. Besides which, even if the tube is empty of air, the mere expansion itself, if it be set up vertically, is work done against gravity.

And if, in any such case, expansion can be altogether stopped by *increasing the external pressure*—fitting an air-tight disk on the tube, for instance, and loading it—it will be found that, *for the same quantity of heat applied*, the temperature given to the substance will be increased. That is to say, a given quantity of heat may either be entirely spent in giving temperature to a mass, or partly in this and partly in doing dynamical work; and, of course, according to our principles, the more it does of one kind of work, the less it can do of the other. If this extra pressure is not sufficient to stop all expansion we may have different proportions of dynamical work and increase of temperature, which complicates the problem.

But we will not pursue this subject into details, but will confine ourselves to one very important branch of it, the most thoroughly worked out and the most easily explained—the elasticity of gases, in which the separation between the thermal and dynamical actions can be readily made.

DYNAMICAL ENERGY OF ELASTIC FLUIDS.

An elastic fluid (gas or vapour) is one which, when confined in any vessel, always fills it and presses against its sides, tending to expand itself farther until met by a pressure equal to its own.

Thus, if we have air in a vertical cylinder closed by a movable piston, it will stand at a certain level when placed in the open air. If we introduce it into the receiver of an air-pump and *exhaust* the air, we relieve it of some of the pressure which kept it down, and the piston will rise; if, on the contrary, we pump more air into the receiver, we increase the pressure, and the piston will sink.

By the 'elasticity' of a particular gas confined in a vessel is meant the degree of force with which it is pressing against the sides, and it is measured, numerically, by the amount of force exerted on each *unit of surface*—square foot, or square inch, &c., as we may agree. This elasticity is known and fixed for every kind of gaseous substance (air, hydrogen, steam, &c.), when we know what the substance is, its temperature, and density.

When we distinguish between 'gases' and 'vapours' we mean by the former word elastic fluids which retain their elasticity within the range of pressure and temperature to which they are ordinarily, in natural circumstances, subjected. By 'vapours' we mean such fluids as are found in ordinary nature to be near the point where they become converted into liquids. Thus steam is a vapour, because a slight cooling will reduce it to water; carbonic acid gas is a gas, because it is only by artificial means that we can reduce it to a liquid or a solid; common air has a still better title to the name of gas, and is described as a 'perfect gas,' because no means have yet been devised for reducing it to another state.

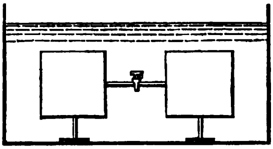
As has been already stated, most gases have been proved to be, like steam and carbonic acid gas, capable of existing as a liquid or solid; and it is the *work of heat* to convert them from solid to liquid, and from liquid to vapour and gas. Up to a certain point heat applied to a liquid increases its temperature, usually expanding it somewhat in bulk; after this point it ceases to give it temperature, but expands it and gives it the gaseous character—the power of expanding against pressure.

And whether it be true or not that this is the origin of all gases, at any rate, heat applied to any gaseous liquid confined in a given vessel does invariably increase both its temperature and its pressure against the sides of the vessel; that is, it gives it capacity both to warm other bodies and to move them. Therefore, according to our principles, either of these actions will require an expenditure of energy. If the gas is allowed to do work on the piston, it must either lose temperature, or elasticity (resume or come nearer to its condition as a liquid), or do both. And so it does.

Elasticity does dynamical work in the receiver of an air-pump when it is being exhausted. The action is not like that of the common water-pump, in which, when the pressure on the surface of part of the water in the well is relieved by making a vacuum above it, the air—an external agent—exerts force and pushes the water up the pipe. It is the air in the receiver itself which in some way causes a portion of it to rush violently out at the valve, giving to this portion a sensible kinetic energy. And accordingly, it has long been known (though this explanation was not thought of till recently) that *cold* is produced; that is, heat disappears. We must now look out for some manageable shape of experiment to enable us to measure the quantities concerned.

The action is obviously precisely the same when air compressed in a vessel, so as to be denser than the surrounding air, is then allowed to rush out by turning a stopcock. Accordingly, the vessel is found to be cooled, and the quantity of heat lost may be ascertained by keeping the vessel immersed in a water-bath, and seeing how much water it can cool by 1°.

But in this (an old experiment) we take no account of what happens to the air that escapes.



Mr. Joule varied the experiment by having two vessels, connected by a pipe and stopcock. In one the air was very much compressed; in the other, it was in its usual state of atmospheric pressure. They can be either placed each in a separate water-bath, provided with a thermometer, or both in one.

When the communication is opened air rushes from one vessel to another. If the cock be closed before the density has become uniform in both, the air that was in violent motion in the second vessel almost immediately comes to rest. Kinetic energy, which had been produced at the expense of heat (as we saw), has now disappeared, and therefore, according to our previous investigations, heat should appear in its place. And it does so. When the two vessels are in separate baths the first is found to have cooled the water, and the second to have heated it. When they are in the same bath no perceptible effect is produced: the heating and cooling are almost, if not quite, equal.

When experiments, the same in principle as these, are made with extraordinary care on some gaseous bodies—carbonic acid, for instance, or heated steam—it is found that the final result is a slight cooling. But with common air, hydrogen, and generally those gases which are most removed from that state of pressure and cold in which they may be expected to become fluid, the difference, if any, is insensible to all present tests, or very nearly so.

That in the whole process some heat should be lost is not extraordinary, for the gas is not in the same state at the beginning and end of the operation. It has expanded itself. And we merely conclude that in so doing it has expended some of its energy; that is, that it cannot go on expanding itself indefinitely, but will lose its elasticity, unless heat be supplied to it.

But the experiments show that with the more permanent and, as they are called, 'perfect gases,' within the limits which have been observed, mere expansion *costs nothing measurable*, any more than horizontal displacement on a perfectly smooth surface, or alteration of shape in a 'perfect fluid' or 'perfectly flexible string': motion, and consequent kinetic energy, must, it is true, be given to the masses in order to alter their shape and size; and this costs something; but either this remains as *motion*, or else, if destroyed by friction or impact, it gives equivalent heat to the system.

If, then, the energy of a perfect gas, such as air, instead of being allowed to give itself kinetic energy, as in the last experiment, be employed to raise a weight with a steady slow pressure, we ought in this case also to find a disappearance or expenditure of heat exactly equivalent to the foot-pound work done.

The following facts and calculations will prove that it is so:—

If air confined in a cylinder with the piston clamped down be heated, it is clear that the heat does nothing but warm the air. If the piston be unclamped and allowed to adjust itself to the pressure of the air in the interior, and heat be then applied, the elasticity will be gradually increased, the air will expand, raising the piston against its own weight and against the pressure of the exterior atmosphere; the applied heat thus at the same time giving temperature to the air, and doing foot-pound work. The heat required in this case to raise the mass of air by 1° ought to exceed the quantity necessary in the first case by the numerical quantity $= \frac{\text{work done in foot-pounds.}}{772}$ The numbers which have to be taken from experimental data are variously given in different books of authority, the necessary experiments being of a very delicate nature, and so liable to slight errors. A selection might be made of them bringing out the result we expect with a very marvellous accuracy. The *data* here used are taken without any aim at giving a greater appearance of certainty than the facts seem to warrant. It is well that the student should have the opportunity of seeing that physical evidence is, after all, not quite the same thing as mathematical reasoning and calculation. Our certitude in such matters is gradually acquired by seeing how experiments more and more carefully made come closer together in their results, and at the same time fit in more and more with the theory.

If we have 1 lb. of air in a cylinder, with a base one foot square, and with the ordinary pressure of the air upon it (taking no account of the weight of the piston itself, which may be made as thin as we please), and at the temperature of freezing (32° F.), the height at which the piston will stand will be 12·393 feet; in other words, the weight of 12·393 cubic feet of air under these circumstances is 1 lb. Air, under a given pressure, expands by about $\frac{1}{491}$th of its volume for every rise of 1° F. (·03665 for 1° Cent. Regnault, in Jamin). Therefore, when heated to 33° the piston will have risen $\frac{12{\cdot}393}{491}$ ft. The pressure on the piston, in these circumstances, is 2,116 lbs. Therefore the foot-pound work done is $\frac{2116 \times 12{\cdot}393}{491}$, and the equivalent heat should be $\frac{2116 \times 12{\cdot}393}{491 \times 772}$ (A)

On the other hand, the quantity of heat necessary to raise 1 lb. of air under constant pressure by 1° F. is found to be 0·23741 (Regnault, in Jamin). And the quantity necessary to produce the same rise of temperature when no expansion is allowed is found, by experiment and calculations having no reference to this theory, to be $\frac{1}{1{\cdot}408}$th of this, or $\frac{{\cdot}23741}{1{\cdot}408}$. The difference between these two quantities is $\frac{408}{1408} \times {\cdot}23741$. And this difference, we have seen (p. 89), ought to be equal to the quantity (A).

Actual calculation brings them out respectively about 0·0692, and 0·0688, the difference being $\frac{4}{668}$ or $\frac{1}{167}$ of the smaller quantity—quite as close as can be reasonably expected.

In our typical series the steam-engine was taken as the example of dynamical work done by the elastic force of vapour, dynamical energy being also the origin of the vaporisation of the water. The action here (at least when the steam is passed direct from the boiler to the cylinder) is less simple, the behaviour of steam not being quite the same as that of gas. And nice measurements of work done are not easy in working machinery. Nevertheless, the attempt has been made by M. Hirn, an engineer of Alsace, with results roughly agreeing with theory. We will here, assuming the equivalence of heat and work to be established, point out how the efficiency of a steam-engine must be measured, and in what the main difference between steam and air consists.

When water, kept at a fixed temperature, gives out steam into a limited chamber in free communication with it until it has filled the chamber, the steam and water must, like all other bodies in contact, have the same temperature. And it is found that when the steam has attained a certain density, greater the higher the temperature maintained, evaporation ceases. If heat be added, both water and steam rise in temperature; and then, of course, more evaporation takes place, in accordance with the above statement. If, on the other hand, we try to condense the steam by pressure, some of it passes into water again (see p. 79), giving out heat in the process, and a readjustment, in accordance with our statement, again takes place. But for a given temperature there is an ascertained given density of the steam evaporated which can be reached but can never be exceeded, and which is known as 'the density of *saturated* steam,' for that temperature. And steam of a given density and temperature, as has been stated (p. 85), has a given and known elasticity. All these connected quantities have long ago been recorded and tabulated for practical use.

We can, then, measure with little difficulty exactly how much thermal energy is sent into the cylinder from the boiler per second or minute. This steam is let in, alternately, on one side and the other of the piston, driving it backwards and forwards, and so working the machinery; and at the end of each stroke that which entered and did its work, and so is weakened in its elastic force, is put into communication with, and partly expands and is partly driven by the return stroke, into a *condenser*, which we may here suppose to be a coil passing through cold water. The quantity of heat thus brought out of the cylinder is easily ascertainable, as in pp. 80, 81. The difference between the thermal energy brought in and finally expelled is, then, what has worked the engine. The work will consist partly of overcoming friction (with production of corresponding heat), and partly of motion given to wheels, &c., and partly of external work—pumping, &c. The sum total, reduced to foot-pounds, should be 772 × the difference of the incoming and outgoing quantities of heat.

It used to be taken for granted that what passed out of the cylinder into the condenser is *steam*. And, having tables of the quantity of heat represented by a given quantity of steam of given elasticity, it was thought sufficient to measure the elasticity of the vapour as it passes out of the cylinder by letting it press on its passage against a spring and noting how far it moved it, or some similar contrivance; and then, from this observation of the elasticity and from the known quantity of steam that passed from the boiler, to *calculate* the heat lost, instead of *measuring* it as above explained. This would have been a correct method, had the substance used been a heated gas, like air. But the fact is that *saturated steam*, when it expands, doing work against a pressure, does not at once become cooled, but part of it returns to the state of water (see p. 86); the heat which this transformation evolves, which would heat surrounding water, here supplies the energy necessary to enable the remaining steam to do the work on the piston, and so what comes out into the condenser is a mixture of water and steam of equal temperature, or thereabouts. The old form of engine is, therefore, much more effective —does more work with a given consumption of heat in the furnace—than was at first calculated. The more modern practice, or that which is becoming more common, is to *superheat* the steam—heat it in a chamber after it is cut off from the boiler—and then pass it into the cylinder. It then has the properties of a gas, and the inconvenience of the formation of water is avoided.

ELECTRIC ENERGY.

In our typical series Electricity is introduced as a temporary state of the conductor (or, it may be, of the air, &c. surrounding and intervening between the parts of the apparatus), leading on to the production of heat or chemical change: much as the temporary stretching of the string, in our dynamical illustrations, passes the force on, as it were, from the hand, or from one weight to another weight. In both cases, at the end of the operation, the intermediate agent, the electrified body or the stretched string, &c., returns to its original state, and we have only to compare the prime mover and the final result. The illustration already given (p. 75 b) is sufficient to make the principle clear to the student, where heat alone is the final outcome. The case of chemical decomposition partially taking the place of heat will be treated in the next section.

And these are, in truth, the more important aspects in which this very imperfectly understood agent presents itself in nature. The outcome of the thunderstorm is mainly in the flash of lightning, which is a sudden passing into heat of a previously electrified state of clouds, earth, and intermediate air—the attractions and repulsions which have been made so useful in telegraphy are but bye-incidents, so to say, in a process which may be described as the conduction of chemical action or its result to a distance. At all events, neither the amount of experimental knowledge nor the mathematical analysis which has been applied to explain and connect the facts are such as appear to bring the science of Electricity within the proper scope of this treatise. We pass on, therefore, to chemical decomposition and recomposition, including something of the connection of these processes with galvanic electricity;

only observing that if electric effects *remain* at the end of an experiment or are produced beyond the range of the particular system we are observing, we must allow for this as work done, in summing up our results.

CHEMICAL ENERGY.

The discovery that *Quantity of Heat* ranks, as a numerically measurable thing, with dynamical work done, has made the production and expenditure of heat the central phenomenon which connects all branches of physical science. It is not always so much the facts that are new in each science (though the new theory has often led to new experiments), as the relation in which they are found to stand to other facts.

It is not new, for instance, that chemical action, the composition and decomposition of elementary substances, sometimes evolves much more heat than what (if any) may be required to start the action (which is what we see whenever we light a fire), nor that, in other cases, great heat has to be supplied from external sources in order to carry on the action at all. Nor yet is it new that chemical action is one of the main sources, directly or indirectly, of motion: the combustion of coal is a chemical action, and works the steam-engine; the explosion of gunpowder is another, and fires the cannon; and, as has been mentioned, animal energy has its source in chemical change within the body.

But the connection between these facts was not seen; and each experiment was often made, and the result noted, without attending to the reverse effect when the experiment is inverted, or the calculation of the compound effect when several processes went on together. We are now, by our doctrine, enabled to see much of what *ought to be*, and to test our expectation according to circumstances.

In general, where two chemical elements, or compounds, *combine* to form a new one, without being accompanied by any other phenomenon, heat is evolved ; even though, as is very commonly the case, no action at all takes place without the preliminary application of *some heat*. Thus, charcoal is almost pure 'carbon'—one of the elementary substances. Now, it will remain unchanged as long as we please in air of the ordinary temperature. But if we apply a lighted match to one end of a piece, and once *kindle it*, it will *burn*—that is, unite itself chemically with the oxygen of the air, evolving heat—and this combination will form a gas, carbonic acid gas. The quantity of heat evolved in this process can be measured thus:—Let the carbon, first lit at one end, be placed in a vessel, itself surrounded with water ; let pure oxygen be supplied through a pipe, and let the gas produced pass out through another pipe, led from the top of the vessel, into the surrounding water in coils of sufficient length to ensure its being cooled down to the common temperature before it emerges into the air again—an apparatus closely resembling the still (in p. 80), only with the burning charcoal taking the place of the boiling water. Then we can measure how much oxygen we supply, and can also see how much heat is given out to the water for any given weight of carbon burnt into carbonic acid gas. It is found that in 44 lbs., ozs., or grains, &c. of carbonic acid gas there are 12 parts of carbon to 32 of oxygen, and that the quantity of heat evolved is about 96,960 units, or 2,203 units to the lb. of gas produced, or 8,080 to the 1 lb. of carbon burnt.

If, then, our doctrine be at all applicable to chemical change, the combustion of 1 lb. of carbon, when it is so carried on as to result in the production of this gas, reduced to the original temperature of the materials used, and with no dynamical or other effect simultaneously produced, ranks as *work done* to the numerical amount of 8,080 units of heat: which, again, is equivalent to 772 × 8,080 foot-pound units of work.

If the gas, in the course of its formation, does work—for example, if it is collected in a cylinder exhausted of its air, and expands by lifting a piston—the heat measured will, of course, be less by the work done—area of piston in feet × 2,116 (pp. 89, 90)—and also by the small expenditure necessary for expanding the gas itself (pp. 87, 88).

On the other hand, to reduce the gas back to its original constituents in their original state should require the application and disappearance of 96,960 units of heat, or some equivalent energy.

Supposing all this true, we should farther infer that for every 1 lb. of carbon which has entered into any chemical combinations and actions, starting *in its simple state*, passing through any number of intermediate states, but appearing at the end of the operations as in composition with oxygen *in the state of carbonic acid gas*, 8,080 × 772 units of energy, measured on the foot-pound scale, should appear somewhere, within this system of materials or externally to them, as heat or work, *in addition to the results of all the other combinations and operations*, as the result of the *exhaustion of the energy* which was stored up in the carbon and oxygen. Similar experiments have been made with most other chemical elements and the several states of combination in which they may appear. Thus, the combustion of 1 lb. of hydrogen gas—the lightest gas known to us, and about fifteen times as light as common air—combining with 8 lbs. of oxygen, evolves 34,462 units of heat.

All our common fuels and sources of light—wood, oils, tallow, &c.—are compounds mainly of carbon and hydrogen. But, in estimating the amount of heat which may be got from them by burning them thoroughly into carbonic acid gas and water, we must observe that simple carbon and simple hydrogen will lose some of their energy in undergoing the preliminary process of passing from that simple state to that of wood, oil, &c.; and we should therefore subtract this amount from the total arrived at on the above principles.

There are, apparently, some cases of combination which, instead of yielding heat, require and consume heat in their *formation*; but this makes no difference in the principles here expounded. They will give out the heat again in their *decomposition.**

These are the consequences we should draw from the experimental facts above stated or alluded to, on the supposition that the doctrine of Energy applies to these actions. We must now give some indications of the tests to which the theory has been subjected, and show how far they confirm its truth.

Direct experiments to ascertain how much heat must be supplied from external sources, *and must be spent* in decomposing, say, water or carbonic acid gas, are not easily made. It is comparatively easy to measure all the heat that comes out of an apparatus such as above described, with water completely surrounding it; but not so easy to ensure that *all* the heat produced by a spirit-lamp, a candle, or heated coals *goes into* a vessel where our materials are undergoing decomposition; so that, though we may have ascertained very exactly how much heat we have thus created, we are still at a loss for any accurate measure of what we have so spent. The nature of the experiments that can be made is of the following kind:—

We find, as above, that the combination of 1 lb. of hydrogen with 8 lbs. of oxygen forms 9 lbs. of water with evolution of, in round numbers, 34,000 units of heat:

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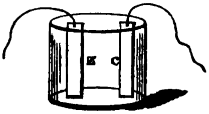
* Jamin, *Cours de Physique*, II. p. 421 (1859), mentions protoxide of nitrogen and peroxide of hydrogen.

similarly we find that 8 lbs. of oxygen combining with 33 lbs. of zinc form an oxide of zinc, with evolution of 53,000 units of heat.

If, then, we can enclose in one experimental vessel 9 parts by weight of water with 33 parts by weight of zinc, and can get these substances to act chemically on each other, so that the water shall part with its oxygen to the zinc, leaving as the result 1 lb. of hydrogen and 41 lbs. of oxide of zinc, the first operation is equivalent, as work done, to the spending of 34,000 proportional parts of heat; and the second to the evolution of 53,000; and the result ought to be a heating of the surrounding water in the proportion indicated by the *difference*, or 19,000 parts (that is, if lbs. of the materials are used, then 19,000 units of heat should appear; if grains, then the $\frac{1}{7,000}$ part of this). And any number of problems of this kind can be treated as tests. Experiments of this kind, with fairly satisfactory results, have been made.

The method of decomposition of water by an 'electric current,' exhibited in our typical series, might theoretically be conceived to be used for ascertaining how much heat, or other energy, goes to the operation. Letting alone all the previous operations, and starting with the steam-engine, we should have to ascertain what the expenditure is there (pp. 91, 92); and then, if we could enclose the whole apparatus on which the engine acts in a calorimeter, the difference between the whole heat evolved and that due to the expenditure in the engine would be what had gone to the decomposition of whatever water should be found to have disappeared, vanishing into hydrogen and oxygen gases. And a modification of the process, making the apparatus much more manageable, really enables us to make measurements of this kind, and will also extend the student's acquaintance with electrical action—with 'galvanism,' or 'current electricity.'



If a bar of very pure zinc and a strip of copper be placed in a glass of water mixed with some sulphuric acid, without touching, and with a copper wire fastened to the top of each piece of metal, the ends of which can be brought into contact or used as in (pp. 6, 7, *note*); then, while the wires are kept separate, it will be seen that the materials are in an unusual state. Bubbles of hydrogen will be seen formed on and adhering to the zinc, and the wires will, by delicate tests, be found to be electrified.* But no continued action will be going on. It is like the water in the reservoir pressing against the sluice-gate and oozing through its chinks, waiting to rush down when a way is opened for it. There is stored up energy in the system, all but acting. That energy we know to be stored up in the capacity of the zinc and oxygen to combine (p. 98); and, if action takes place, the work done by it will be the decomposition of water, with a balance to come out, in heat, or in some farther action to which we may be able to set it, at the rate of between 18,000 and 19,000 thermal units for every 33 lbs. of zinc consumed and every 1 lb. of hydrogen set free.† Meantime, there appears to be a nicely adjusted, easily disturbed balance between this tendency to chemical action, the existing hold that the hydrogen has on the oxygen, and a certain *strain* in the whole field of the apparatus, manifested by the electric condition of the wires.

If now the *circuit* be *closed*—that is, if the ends of the wires be made to touch each other and act as a single connecting wire—the resistance to action, whatever be its seat, gives way; the zinc and oxygen begin to unite, and the hydrogen is set free and escapes into the air. And if the whole apparatus works in a calorimeter the amount of heat produced over the whole, partly in the 'cell' (where the chemical action goes on), partly in the wires, is found to agree pretty closely with what our theory requires.

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* That which comes from the zinc like rubbed resin, or 'negatively;' that which comes from the copper like rubbed glass, or 'positively.'

† There seems ground for believing that the action is not really so simple as this, the old-fashioned explanation, implies. It is probable that the zinc takes its oxygen direct from the sulphuric acid, and that it is the residuary part of this chemical compound which thus robs the water of its oxygen, and so relieves the hydrogen. But our illustration is not substantially affected by accepting one view or the other.

Numerous other kinds of 'galvanic cells' have been devised, in which the chemical elements combining and decomposing are different from the above. And the resulting heat has always been found satisfactorily to agree with what would be anticipated from direct experiments in the same elements in the ordinary laboratory; thus confirming our doctrine that, in whatever way results may be brought about, the first and the final state of the system are sufficient to determine the quantity of energy expended.

The connection and numerical relation between the amount of work done, per second or other unit of time, in such a galvanic cell, on one hand, and the intensity of the electric strain in the parts on the other, is matter for treatises on electricity. But there are some circumstances of the action which both illustrate and are illustrated by the doctrine of energy, which also connect this experiment with the one already described (pp. 6, 7); and it may be profitable to go into some details here on the subject.

We may see, first, that the whole 'circuit' is concerned in the motion.

1. The connecting wire is thrown into a new condition —that, namely, which is now familiar to all, of the telegraph wire, capable of deflecting a magnet from its natural position, and of turning soft iron into a temporary magnet.

2. Neither the chemical composition and decomposition

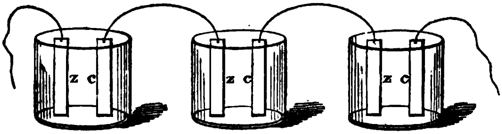
nor the evolution of the balance of heat take place in the ordinary way, as in the laboratory. If zinc decomposes water without the intervention of these arrangements, where it takes up the oxygen, there it liberates the hydrogen which was united with it; and there, also, the heat is produced, which may afterwards be conducted or radiated to other places according to the laws which govern the diffusion of heat. But in this experiment the oxygen is seized by the zinc at its own surface, and the corresponding hydrogen is set free *at the surface of the copper*—it may be several inches off:—and the heat, though always, as we have seen, the same in quantity as in the laboratory, is produced, *immediately* and not by diffusion, partly in the cell and partly in the wire, in proportions determined by laws specially belonging to electric science to expound.

If we try to represent to ourselves the kind of agency at work, we seem to see the oxygen pulled or pushed towards the zinc, and the hydrogen towards the copper—the one to unite with the corresponding metal, the other to escape free—while the metallic connections, which do not participate in this motion, are necessary to the action as receiving and transmitting stresses or strains between the two 'poles' or extremities of the line of liquid which is being torn asunder; much as a cord or a rod acts in a machine.

Now, the wires in the former experiment (pp. 6, 7), when introduced into the acidulated water, are found to be in this same state, as tested by their power to deflect a magnet, &c.

We may, therefore, expect that if, in the present case, we again separate the wires and plunge them into such a mixture, the water will be decomposed. And it is found in fact that, with some of the galvanic cells above alluded to, this is the case. But it is not so with the particular combination we have been describing in detail: the apparatus remains in the same quiescent state, with the wires dipped in the liquid as while they lay on the table.

If, however, instead of a single galvanic cell of this description, we employ a 'galvanic battery' of several cells connected as in the figure annexed, the copper of the first cell being joined by a wire to the zinc of the next, and so on, the first zinc and the last copper taking the place of those in our single cell, a sufficient number of such cells will work, and decompose the water in the electrolytic cell. Can we in any degree explain the difference in these cases by our principles?



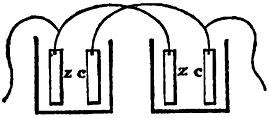
Take first the single cell, with a single wire connecting the copper and zinc. If our conception of the part played by the wire be correct—viz., that it is analogous to that of a stretched string or compressed rod in machinery—we must suppose the strain at one, say the copper end, transmitted without change to the zinc end, and there balanced by an equal counter-strain through the zinc; and if we actually or ideally cut the wires in two and keep them in contact, we must conceive (as in the case of a rod) each end pushing and pushed by the other end, with equal and opposite reaction, and still transmitting the original pressure. If then, instead of being held in contact, they are plunged in the liquid, *and if any action takes place at all*, we should expect it to represent and be equal to the *same pull or push* which immediately takes place in the galvanic cell. In other words, we may anticipate that, as decomposition of water is the work done in both places, the *same amount* of water will be decomposed in the galvanic and in the 'electrolytic' cell, or 'voltameter' (as it is called).

Whether this reasoning be well-founded or not, such is the law experimentally found to govern all galvanic action; *in every part of the system* where, by this agency, some chemical energy (as of the zinc and oxygen) is doing the work of dragging asunder the *same chemical compound* (water here), the *amount* of such work is the *same.*

When different compounds are being decomposed in different parts (as where the electrolytic cell does not contain water, but a 'salt,' say sulphate of potassium), there is a definite chemical *equivalence* between the actions in all parts of the line of action. In the language of chemists, 'the same number of chemical equivalents are decomposed in every galvanic or electrolytic cell.'

Taking this law as established, we see at once why one galvanic cell, of our kind, cannot, and why several combined may decompose water in the electrolytic cell. In the first case, for every pound of water decomposed in the galvanic cell, which we have seen will leave less than 19,000 thermal units of energy to spare, another pound of water must be decomposed in the voltameter. But this would require 34,000 units (p. 96). As these are not forthcoming from any source of energy, no action at all can take place. On the other hand, when there is a battery of several cells, the same quantity of water must, by the law, be decomposed in each; and as in each there are zinc and oxygen combining, and so spending their chemical energy at a rate equivalent to nearly 19,000 thermal units per lb. of zinc in *each galvanic cell*, two such cells at work would suffice to furnish more than 36,000 units; and if *all* this could be evolved in the voltameter, decomposition would go on. As, however, this is not the case, because some of the spare energy from the zinc and oxygen must come out as heat in all parts of the apparatus according to certain known laws, three or four cells must be combined. And, in general, when we know what the source of energy is in any galvanic cell composed of other materials, we can calculate what the spare energy will be, per *chemical equivalent* combined, and what energy is required for each decomposition in the series, and so obtain a condition *without which* no action can take place in the system; though it does not follow that action will necessarily follow whenever this condition is fulfilled.

There is another form of battery sometimes used, in which the cells are said to be joined in 'multiple arc,' which must not be confounded, as to its electrolytic power, with the one just described. The two zinc plates are here coupled together, and a wire conducted from them to make one side of the circuit; and the two copper plates are in like manner joined for the other side.



In this case the two zinc plates, with their wire, really form one *forked* extremity of the circuit; and the copper plates, with their wire, another forked extremity of the *same circuit*, and the strain transmitted through the two final wires into the voltameter represents the joint re-action of the action going on in *both* cells; so that as much water must be decomposed in the voltameter as in the *two* galvanic cells together. In fact, the system is exactly equivalent to one single cell, in which the two zinc plates may be placed side by side and made one plate, and so of the copper.

We may here add, as belonging to our subject, that the energy arising from galvanic action may be applied, not only to the production of heat or of chemical decomposition, as above, but also, like that of steam or gas, to doing direct dynamical work. The deflecting of a magnet is an instance, though on a feeble scale. By other contrivances machines intended to do the work of steam-engines have been constructed, but are found uneconomical. These engines, in which the moving power—answering to the burning coal in the steam-engine—is, in truth, *burning* zinc or some equivalent (zinc, or the like, combining with oxygen)—have been enclosed in a calorimeter, and the amount of heat, showing itself by increase of *temperature*, for every 33 lbs. of zinc consumed, has been found to be less than in the experiment above described by the amount of dynamical work done; just as in the case of the heat found in the steam and water after work done in the cylinder (p. 91).

ANIMAL ENERGY.

Animals, so far as their power of motion and exertion are concerned, may be described as *self-repairing heat engines.* The food necessary for our support contains in it materials for replacing what is daily worn away of our bones, our cartilages, muscles, nerves, and other tissues. But its main bulk is *fuel*—combustible material of much the same composition as wood, coal, oil, &c.—which is *burnt* as it comes in contact with the oxygen of the air breathed in at the lungs and carried in the blood through the whole system of our circulation. This causes the warmth of our bodies, and it also furnishes the energy by the expenditure of which we move our limbs and do external work. What may be the precise *mode* of action —whether more resembling heat of a steam or air-engine, or that of a battery—is a matter for the enquiries of the physiologist, and is probably still uncertain. Dr. Frankland several years ago, in a lecture at the Royal Institution, described it thus, in general terms: 'The combustible food and oxygen co-exist in the blood which courses *through the muscle;* but when the muscle is at rest there is no chemical action between them. A command is sent from the brain to the muscle; the nervous agent'—acting, it would seem, *as a stimulant*—'*determines oxidation;* the *potential becomes active energy,* motion, or heat. Like the piston and cylinder'—in the common heat engines—'the muscle itself is *a machine;* both are subject to wear and tear, but neither contributes in any important degree by its own oxidation to the actual production of the mechanical *power.*'

Whether this be an accurate description, as far as it goes, does not particularly concern our subject. What does concern us is, that experiments have been made in which a man, or a smaller animal, has been confined in a *calorimeter,* and has consumed a certain quantity of food. If no dynamical work was done, the evolution of heat was in some rough correspondence (for the experiment is necessarily a rough one) with the food ultimately consumed and digested; but if such work was done—for instance, a windlass worked and a known weight raised—the amount of heat shown was less by the equivalent of the foot-pound work thus done.*

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* There is an inaccuracy on this subject in Professor Huxley's *Lessons in Elementary Physiology* (4th edition, 1870—Article III., page 2), which it may be worth while to correct, as it will illustrate our subject.

He there supposes a man, not to work a windlass, and so raise a weight, which remains as work to show at the end of the experiment, but to walk up and down within the calorimetrical chamber. And then he sums up, as *separate items* of expenditure, 'exerting mechanical force,' and, *in addition,* 'giving off heat,' which last expenditure alone he supposes to be measured by the melting of the ice. Now, all this 'exertion of mechanical force' is itself spent and converted into heat, as soon as produced, in the stamping on the ground (p. 71, &c.). There is no dynamical energy, potential or kinetic, to show for it at the end of the experiment. So that the ice actually found to be melted is the measure of the *sum* of all the energy of every kind that has come from the man. According as it exceeds or falls short of that

ENERGY OF VEGETATION.

THE food thus burnt and exhausted in the animal economy is derived from vegetables—either directly or in the form of flesh which has, ultimately, come from a vegetable-eating animal—and it is mainly reduced to water, carbonic acid gas, and ammonia. And these are the principal nutriment of vegetables; that is, the materials out of which they build themselves up into the substances which nourish and support animals.

To do this is, clearly, to restore the energy which has been spent by the animal, and, on our principles, this restoration requires the expenditure of some other external energy.

This is supplied by the sun's rays; and, as regards carbon, it seems, almost entirely by those rays which, in the solar spectrum, occupy the middle space: orange, yellow, and green. Fresh green leaves have been placed in tubes of water mixed with carbonic acid gas, and exposed to different parts of the spectrum, and those on which this part of it fell exhibited bubbles of oxygen, the result of the decomposition of the carbonic acid. ('Jamin,' III., p. 513.) There is always carbonic acid in the atmosphere, renewed continually by the animal processes, by combustion, and in other ways, and decomposed again by this process.

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stored up in the food consumed, we may conclude he has left in him less or more energy than he started with.

Professor Huxley uses the phrase, 'exercising *a great amount of force.*' As his treatise is not on dynamics or energy, strict technical language is not to be expected on an incidental point like this. But the student should be warned that the proper phrase is, 'expending a great amount of energy.' 'Force' means the strength of the tendency to induce motion at the moment, and has nothing to do with the amount of work that can be done before exhaustion.

The sun is thus the ultimate source of all the energy which we and all animals are busy in spending. The wood, the oils, &c. which we annually burn; the corn, the grass, the fruits which we or our sheep and cattle eat; and the coal and petroleum, which are the remains of a former vegetation; all are stores of carbon and hydrogen once extracted by his rays through the vegetable machinery from inorganic matter, which we reduce again to that inert state by the chemical processes going on in our bodies while we carry on the functions of life. And it may be added that the sun's heat is also the source of the potential energy of all natural supplies of falling water, by evaporating the sea-water, so giving it elastic power to rise, thence to fall in rain or mist and feed the springs of rivers, &c.

Whence the sun gets its heat—whether it is merely an enormous store of energy, which is being gradually exhausted, or whether there is any discoverable source whence it may be materially derived—is a matter about which there has recently been a great deal of speculation, without, as yet, any satisfactory result.

RECAPITULATION.

We here close our exposition of the elementary and fundamental facts and reasonings which establish the Doctrine of the Conservation of Energy, so far as we have attempted to carry it.

We have first explained the principles and the method by which we measure (*a*) Mass and Force, in connection with and dependence on the Laws of Motion; (*b*) Heat; (*c*) Work Done, in its simplest form, when confined to the transference of a mass from place to place against a resistance, starting from and ending in rest, and with no sensible velocity at any time.

We then gave so much of the mathematical theory of dynamics as was necessary to show the numerical equivalence between Kinetic Energy and work done as thus defined; and were enabled to pass thence to a larger conception and definition of work done by and against forces acting in different directions in a system of bodies in motion, showing how the same measure of kinetic energy still remains in numerical correspondence with the generalised measure of work done.

And we showed that, where all the forces acting fulfil certain conditions, and no other changes take place except dynamical ones, the work done by or against a force represents a corresponding diminution or increase of a capacity for doing more work — that is, a Potential Energy.

We then proceeded to consider generally the cases where Dynamical Energy (kinetic or potential) is spent, with no *dynamical* result left to show, and laid it down as proved by all the best experiments hitherto made (of which we gave samples) that, whenever this happens with no *other* change of a physical character to show for it, heat results in the definite proportion of one thermal unit per 772 foot-pounds spent; and, reciprocally, that heat may, to some extent, be *spent* in producing Dynamical Energy, with the same numerical equivalence.

Finally, we have examined some of the principal classes of changes of a physical and chemical character which occur around us, and have given reasons for believing that the *difference* between any two conditions—as of liquid and solid, or of water and its separated elements, &c.—may always be taken as representing an equivalent difference in Thermal or Dynamical Energy, gained or lost, according to a fixed scale, in some other direction.

So far, then, as observation and experiment have gone, our proposition (p. 3) is proved. The remaining sections of this treatise will deal, first, with an important corollary from our general doctrine as applied to the facts of nature; and then with the question whether any *intelligible* account can be given of that connection between Dynamics and the other physical sciences which has thus been experimentally shown to exist.

DISSIPATION OF ENERGY.

WHATEVER may be the truth about the sun—whether its energy is now replenishing from other sources or not—a consideration of the actual course of nature, in the light of our doctrine of Energy, will tend to show that there is a general tendency to the conversion of all potential energy—dynamical and chemical— into actual or kinetic energy, either motion or heat; and that this tendency must continue, and produce effects, until all collisions and slidings or rollings between the surfaces of masses having different velocities, and all differences of temperature, and all opportunities for chemical action, shall have ceased.

When we say that a system has energy we only mean that there is a certain amount of work which it *can* do, *if the occasion* offers.

Thus, a mass in motion has kinetic energy, measured by $\frac{mv^2}{2}$, because, if it comes to be opposed by any force, it can do work, as measured by Fs, against this force to this amount. But if circumstances are such that no opposing force ever offers itself, it will continue its motion, and retain its energy unimpaired, for ever.

There is, therefore, no contradiction between the two statements, that energy is never destroyed (or, more correctly, that when any particular energy disappears it leaves an equivalent behind it), and that a time must come when there will be no opportunities for changes of energy to take place.

Let us, then, look at what is now going on, and to what it is tending.

(1) If there is such a thing as sensible motion in a perfect vacuum—if, for instance, there is *no matter* between the boundaries of our solar system and the so-called fixed stars—the motion of the sun and stars, moving in accordance with the law of gravitation in ever-varying curves, if they never impinge on one another, may go on for ever. But if it be true, as there is good reason for believing, that the propagation of light from star to star requires a *material fluid medium*, in which vibrations or *waves* can travel (as waves of sound in air), then this motion of the stars must be diminishing. The medium may have the property of *perfect elasticity*, so that no motion is converted into heat (p. 55); nevertheless, the stars, continually altering their velocities by their mutual attractions, will be continually agitating the medium, and parting with some of their motion to it; and the final result would be to allow gravity to bring all the masses together into one, which would then come to rest, or move in unison with the surrounding medium.

(2) Wherever there is impact or sliding between masses not perfectly elastic and perfectly smooth there will be not only passage from sensible motion of the masses, as wholes, into vibratory motion (sensible or not, according to the range of action) in their interior, or communicated to surrounding bodies, but also expenditure and disappearance of dynamical energy, and corresponding production of heat, (p. 71, &c.). So that the final result of impacts and contacts, in the actual universe (if we confine our thoughts to any *finite* portion of it, however large), must be to reduce the motion of all bodies coming in contact to uniformity; with disappearance, on the whole, of

kinetic energy, and the raising the general temperature of everything in the space we are considering. Could that space be made impervious to the passage of heat, the raised temperature would remain as the monument of the kinetic energy that had disappeared. But, as this is impossible, the heat, though at no stage diminished in actual quantity, diffuses itself farther and farther, is *dissipated*, and becomes insensible in any limited space.

(3) Although, as we have seen, heat can be expended in the production of dynamical energy, yet this can only be by virtue of *difference of temperature* in masses that can act on each other: two masses at equal temperature, however much more energy of heat one may have than the other, are not in a condition for acting thermally on each other. Though, therefore, *some* of the heat produced by impact, &c., as above, may be spent in dynamical work, some must remain as generally diffused temperature, ineffective until something colder comes within reach.

(4) Elastic force, also, when allowed to do work, tends to *dissipation*. It can do work against pressures less (per unit of surface) than its own. But, if surrounded by masses resisting pressure, it will spend itself only till it has come down to the general level of pressure. If we imagine unbounded vacuum, with no force like gravity opposing, some gases, we know, and we may reasonably conjecture, all gases, will, at a certain point of expansion, lose their elasticity (p. 88); or, if it be otherwise with any gas, it must at best ultimately fill any limited space, however ample, and so make if it does not meet an equally diffused pressure; or else the particles must go on moving freely, but doing no work, for ever.

(5) Chemical action, we have seen, is of two kinds—one which evolves heat and only requires the presence of the elements in some state admitting the action to begin; the other requiring heat, or some equivalent energy, to be

supplied and *expended* in the action. It is clear that the former actions, if they continue to occur, will ultimately reduce all chemical combinations to such as cannot *change themselves* any farther, and that the heat evolved in this process cannot *adequately* restore the combinations so formed back to their energetic condition, because *some* of the heat arising from combinations will necessarily diffuse itself into space.

Thus, then, we see that in every portion of the universe, however wide a range we give it, dissipation of energy—not expenditure with no return, but the production of a state in which no farther expenditure can take place for want of differences and opposition—is the rule.

MOLECULAR THEORIES.

Newton, who discovered the law of universal gravitation, and showed that its true character is an attractive force between every two particles or portions, great or small, of matter, thus expressed himself in the preface to his great work, the 'Principia,' dated May 8, 1686:—

'Many considerations move me somewhat to suspect that all the phenomena of nature may possibly depend on certain forces, by which the particles of bodies, by virtue of causes as yet unknown, are either driven towards one another and adhere in regular shapes, or else are driven from one another and recede; which forces remaining unknown, philosophers have as yet attempted the investigation of nature without success. But my hope is that these "Principles" will afford some light either to this or to some other and truer manner of philosophising.'*

All subsequent observation and theory has hitherto tended to give strength and definiteness to this suspicion. And the only rational explanation that has been offered of the actual equivalence between dynamical and other changes—of the doctrine of the *Conservation of Energy*—rests on the assumption of the substantial truth of Newton's surmise. The facts remain, whether explained or not; and it may be that the processes of nature are essentially unintelligible to us. But any hypothesis which reduces them to *motions* of invisible and (may be) *inappreciably* small particles, but still particles having size and form, commends itself to us. This treatise shall therefore conclude with an attempt to give the student some definite notion of the theory of the atomic and molecular constitution of substances. He must not look for strict proofs, either experimental or mathematical; only, at most, for indications that some such theory would fit in with and account for physical facts as we observe them.

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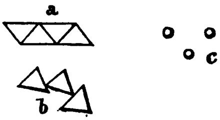
* It will be observed, by those who are familiar with some modern speculations on the subject, that Newton suspects unknown *causes* of these forces or tendencies; but anticipates good fruit whenever a knowledge of the *forces* themselves—their intensities and laws—should be discovered. He did not himself believe that gravity is an *ultimate* fact.

I. It is commonly laid down in elementary books as a primary quality of matter that it is 'impenetrable,' or that it fills the space which it occupies, so as to exclude all other matter while itself is there.

And so the atomic theory (at least as generally accepted) conceives all sensible masses to be resolvable into ultimate, indivisible, impenetrable 'atoms' (Greek for what cannot be cut into smaller parts). But substances as they present themselves to the senses are all penetrable. Take, for instance, water in its fluid state, and suspend some salt in it. The salt will be dissolved; the water will penetrate the solid and the salt the fluid, and the two will be so intermingled, in a fluid condition, that after a time no portion of the mixture can be taken up and examined which shall be more or less salt than the rest. And though water will not dissolve everything, yet there is scarcely any known solid substance which a chemist cannot find means of dissolving in some liquid or other at a proper temperature. Or take two liquids of different colours—say water and brandy. The brandy can, with due precautions, be so poured over the surface of the heavier water as, for the time, to be separated from it by a well-marked horizontal line; yet, although apparently in their proper and natural places—the lighter uppermost—the two liquids will interpenetrate, the water rising against gravity, and the lighter brandy sinking down, until, after a few days perhaps, the mixture will be perfect.

Now, this state of mixture is, so far, intelligible to us, if we conceive each substance, when separate, really to consist of a certain number of exactly similar particles, like masses of very fine dust; whether close-packed (as *a*), touching in points (as *b*), or kept apart at a certain distance (as *c*): provided that we farther suppose them to be so minute that, in the smallest portion of the substance which we can examine by the nicest tests of scientific experiment, the number of such atoms is very large—to be counted by hundreds, thousands, or larger measures still. A perfect mixture will, then, be one in which, in a sample so taken, the proportion of the number of atoms of each kind is *as nearly as possible* the same as in the same mass. For example, if the latter proportion be 2 to 1, and there be 3,005 atoms in one sample, there would be 2,003 of one kind and 1,002 of the other—only one too few of the first kind, the effect of which would be insensible.



If we do not accept this explanation of mixture, as a real mingling of separate particles, we seem driven to admit that *every point* in the fluid is *at once* occupied by something which is both salt (or brandy) *and* water; that is, to give up the notion of impenetrability, or of true individuality of different substances.

It should be observed, however, that this explanation only requires that the mixing substances should in fact separate into distinct particles; not that these particles should under no circumstances be capable of farther division. Our hypothesis explains the fact of mixture, but does not necessarily imply any opinion about the ultimate conceivable state of matter. In strict scientific language 'atoms' mean particles indivisible by any natural means; molecules, portions which keep together in the particular case we are dealing with, but which may be found separated in other circumstances.

II. In both the above cases, and in many others, it is found that the space occupied by the mixture is less than the sum of the spaces the substances occupied when separate. Either, therefore, the atoms have intervals between them (as *b* or *c*), so that the two sets, when mingled, pack close together; or else the single atoms admit of compression and expansion; or both statements might be true at once.

But when we consider the case of *evaporation*—the passage from the liquid to the gaseous state—we can hardly avoid accepting the suggestion that the atoms, in the latter state at all events, are separated, each from each, by an interval of space.

Take water, for instance. When converted into steam at the boiling-point, it occupies nearly 1,700 times the space it did as water. And whereas in its first state, with its particles (it may be) in close contact, there is no disposition to expand, but rather, as we shall presently see, an actual *cohesion* of the parts; as steam, on the other hand, there is in it a striving to expand farther and farther as more and more room is given to it.

Moreover, this disposition is not *permanently* attached to the atoms; but, as we have seen, depends entirely upon the heat imparted to the mass.

And within the space thus occupied by steam we may introduce gas after gas—either fresh doses of the same substances or of other substances—each pressing outwards, *just as much as if it were alone in the chamber*, with no limit as to quantity except this—that when the total pressure against the wall of the chamber is that at which steam would, at that particular temperature, begin to condense (p. 91), it will do so here; and so of the other substances. But, on the application of more heat, they will be reconverted into vapour.

Once admit that we may speculate at all about *how* all this can be, and one does not see that any other conclusion can be come to but that gases consist of particles, with comparatively large spaces between them.

III. And now come the questions—In what state do these separate particles exist? in motion, or at rest? exerting force on each other, attractive or repulsive, or inert? And what light do these phenomena throw on the nature of heat and chemical action?

That in the solid state the particles of a substance exert force on each other, is almost too clear to need pointing out; their presence both prevents and causes motion in their neighbours. If we pull the two ends of a stick in opposite ways, the *cohesion* of the parts prevents anything more than (usually) a very small expansion and separation to take place; if we only apply the force at one end, that same cohesion (that is, attraction) pulls the other end after it. And so, when we push instead of pulling, a slight compression is the utmost we can produce. Only, in each case, if the forces applied be very great, we may overcome the internal ones, and break or crush the stick.

In the case of liquids, the force which the particles of a homogeneous substance exert is only, or chiefly, sensible at the surface. The cohesion of water is proved by its gathering in round drops on cabbage-leaves, or hanging from roof-eaves, or momentarily maintaining itself as bubbles, &c. In all these instances the force of gravity tends to bring the whole mass down to the lowest level; and the force of cohesion (aided in the case of eavesdrops by a farther attraction between the water and the roof to which it clings) prevents the motion. No doubt the same forces are acting between neighbouring particles in the interior of the fluid; but, as there is the same force acting on all sides round each particle there, they do not produce the like sensible effect in causing or preventing motion.

As regards substances of differing kinds, the phenomena of *mixture*, which we have above described, seem sufficiently to prove the action of mutual forces. The salt, for instance, while surrounded by dry air, will remain cohering together; but when immersed in water its parts will move in all directions through the interstices of the matter that surrounds it. Either, then, the air prevents some natural tendency to motion in the particles of salt, or the water causes such a tendency. So of the brandy and water. Without the brandy superimposed the water, if in an open vessel, will in time evaporate; so that here there does seem to be some tendency to motion from internal forces, independent of the brandy. But in a cool atmosphere the effect of evaporation in diminishing the bulk of the water may be insensible for an almost indefinite time; whereas, the presence of the brandy causes a sensible rise, in a very short time, of a quantity which it will thenceforth *retain* there, allowing evaporation to go on, much as before, at the upper surface of the mixed fluid.

IV. In the gaseous state, instead of any appearance of cohesion we see only a tendency to indefinite expansion.

And we have seen that this is not due to a permanent force between the particles, depending on the distance, but is entirely the result of external work done upon the parts of the gas; and in permanent gases, and in all gases when sufficiently heated, is directly proportional to the amount of work done on it—on the expenditure, that is, of heat or an equivalent dynamical energy upon it.

And if we examine into other effects of thermal energy, made to do work on a substance, we find them analogous to this. Its ordinary effect, besides increasing temperature, is to cause expansion (p. 84); that is, to do work against the cohesive forces of the molecules. At some certain temperature, different for each substance, it ceases to increase the temperature of *solids*, and is *entirely* spent in *liquefying* them; that is, in relaxing, almost to the point of neutralising, the efficacy of the cohesive forces. In all these three cases part or the whole of the applied energy produces an effect antagonistic or opposite to the tendency of the attractive forces.

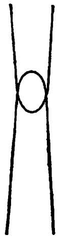
Now, if we are to explain natural phenomena by reducing them to the effects of forces on atoms, *external work* done on a substance can only mean giving motion, and, consequently, kinetic energy, to it; either visible motion of the whole or its larger parts, or *molecular motion* among its ultimate particles. We have, then, to see whether motion communicated from without to the internal molecules of a substance can be conceived to be capable of, first, doing work against; secondly, neutralising the mutual internal forces; and, thirdly, of giving to the *separate molecules* (see p. 117) an independent tendency to fly apart. If so, then we may suppose *heating* a substance to mean 'giving some kind of motion to its smallest parts.'

V. As we now know experimentally that all these changes in the condition of substances are subject to the law of Conservation of Energy, we may most naturally start with supposing that the forces which the particles mutually exert are of a permanent character—that is, are the same whenever the relative positions are the same, though varying by some law when the distances vary (p. 60).

Moreover, in liquids and solids, in their ordinary condition, we find *cohesion* to be the prevailing characteristic (p. 118); that is, at the ordinary distances the forces are attractive; though the enormous resistance to compression may seem to indicate that repulsive forces soon come into play when the distances are reduced.*

Let us then see what would be some of the general characteristics of the mutual actions of systems—such as clusters of stars are on a large scale—consisting of moving masses mutually attracting one another with forces rapidly diminishing as their average distances increase; and compare them with what we know experimentally of internal changes.

(1) Two bodies subject only to their mutual attractions, if their motions at any one particular moment are not in the same straight line, will never come in contact, but will move round one another in some curved *orbit*. (The proof of this is quite elementary, but would be out of place here. It is Newton's First Proposition.) The orbit may be, like a comet's, very nearly a straight line in the greater part of it, turning sharp round at each extremity—at the most distant with very sluggish and at the nearest with enormous impetus.



This shows that what we call elasticity in a mass *may*, wholly or in part, be the result of *attractive* force combined with motion. A blow given on the surface of a solid mass drives the particles inwards; but the result may be a pirouette round some of their inner neighbours, and an equally strong outward impetus, driving back the hammer with an energy proportionate to that which it had given.

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* The generally received molecular theory of gases (see Maxwell's *Heat*) supposes perfect elasticity between ultimate molecules in their closest contact, both among themselves and with the molecules of the solid containing vessel.

(2) If an increase of *relative* velocity be given, by any external agency, to the two bodies, whether when receding from or approaching each other, the result will be to *enlarge* the dimensions of the orbit; the force being less able at each point to check the motion and bring the body round. If, on the other hand, the relative velocity be diminished, the orbit will be *contracted.*

If the velocity be increased beyond a certain limit, the result will be that the utmost work that can be done by the forces (p. 64) will not suffice to overcome the kinetic energy of the bodies and bring them nearer again; and, while short of this limit, the increased velocity may separate them so far that one or other of them may come under the stronger influence of another body, and so part company with the first and become a member of another system: as it may be that some comets have only once come into our solar system, and have since wandered on from sun to sun, until perhaps they may have become fixed by one of them.

This suggests a theory of Vaporisation.

When energy is thrown into a system by dynamical action, or by direct heating, if the outer-lying particles thence acquire greater velocities, they may fly off, escaping the bonds in which they have been held by their neighbours; and a set of molecules thus set free, and wandering in open space, or bounding and rebounding in a confined chamber, may be the *gaseous state.* When one such free molecule approaches another in the course of their respective motions their mutual attractive force will be as little able to keep them together as were the internal forces of the substance from which they have escaped; they will *swerve*, more or less, from their courses, and go on. Only, to complete the theory, we must suppose that, if they actually meet and clash they behave as perfectly elastic bodies, and rebound with their original energy. And we must conceive the rebound from the sides of any chamber within which they may be confined as of this same character.

This explanation of the gaseous state falls in with the dynamical phenomena (p. 85). The tendency to indefinite expansion is merely the pursuing by each molecule of the course on which it started, in accordance with the First Law of Motion: the pressure on the enclosing vessel is the effect of the continued impulses of the moving masses, as of billiard-balls against the cushion. If the sides of the vessel remain fixed the rebound is with the same velocity as the impact; but if the vessel yields (as when one side is a movable piston) as much kinetic energy is lost by the moving particles as is measured by the work done on the piston. And so the whole series of effects has been followed out and explained with great probability. But the student must seek for details in works specially devoted to *Thermodynamics*.

(3) If, instead of two masses only, we consider a cluster, forming such a system as we see in a solid or liquid mass of sensibly *homogeneous constitution*—that is, with the same average size, force, and intervals between the atoms and the molecules, in all parts—although the actual effect on any particular particle produced by an addition made to the *average* velocity of the whole may not be calculable by our mathematical resources, yet we can, in a general way, see that, for the distribution of the atoms to remain substantially of the same character, the interspaces must become greater in some proportion with the increase of velocities; the same kind of *balance* of force and velocity must prevail, or the particles would be drawn away from their old orbits and neighbours, to rearrange themselves in some new combination, which would be equivalent to what we have called assuming a new *physical condition*.

This suggests an explanation of the ordinary effect of heating, namely, *combined* increase of temperature and expansion, unless expansion is prevented by increased external pressure, in which case the temperature is farther raised. In this latter case all the external *energy* spent comes out in velocity given to the parts, while the reaction of the increased external *pressure* keeps them in the same orbits: in the former case the orbits expand until they reach the dimensions corresponding to the new average velocity.

To follow up this subject is the business of other treatises; but a few words may be added, in farther explanation of the commonly received theory.

The special action by which the energy of a body hotter than its neighbours manifests itself is the immediate diffusion of temperature all around, not only to bodies in contact, but to those separated by apparently empty space. This latter fact, and the connection established between heat and light, have led to the conclusion that there is a subtle medium—otherwise insensible and undetected by ordinary dynamical effects—permeating all vacant space, probably even the interstices between the atoms of other substances—the undulations of which produce the sensation of heat. In a space where all the molecular motions of different substances are such as to communicate equal undulatory motions to the parts of this medium in the same time, as much motion will be received as given by each molecule, and this represents the case of 'equally diffused temperatures.' If one mass has less than its proper motion, it will receive from the medium more than it gives, and expand and rise in temperature; if it has more, it will give out motion, contract, and lose heat.

On this theory we can clearly see the difference between *temperature* and *quantity of heat.* If a well-made boat and a square box of equal mass are placed in water, and set going with the same velocity, each has the same kinetic energy, and by the time that each has come to rest it will have communicated the same amount of motion—kinetic energy—to the water. But the boat will keep much longer in motion than the box, and will, at each moment, be producing much less commotion in the water. The excess of kinetic energy over that which would keep the mass, or the molecule, moving exactly in unison with the surrounding medium represents the excess of *heat* which it must get rid of before relative rest be attained; while the amount of ripple or vibration it is at the moment communicating to the medium represents the comparative temperature.

Note.—The electric condition has this in common with the heated condition, that it immediately affects distant bodies, putting them into an analogous (not an identical, but rather an opposite) state. The most recent speculation about the nature of this condition is, that it consists, mainly or wholly, in a *strain*—an alteration in the potential energy—between the molecules of this same medium, to which the name 'Ether' is commonly given.

VI. When the work done on a substance has increased both its kinetic and its potential energy, and the former is gradually dissipated as heat, the latter will in general be called into action, and to some extent spent in producing additional kinetic energy, which will again become a farther source of heat; so that, ultimately, though not immediately, *all* the energy, of either kind, originally imparted may come out as heat. This power of producing heat by relapsing into its first condition is what used to be called the *latent heat* of the substance in its actual condition. It might be called its potential heat.

VII. Let us next consider the application of the molecular theory to the case of chemical combination.

If we take hydrogen and oxygen gas at the same temperature and of such densities respectively that they exert the same pressure (per square inch) on the sides of the vessels containing them—as may be tested by closing their tops by equal movable pistons (see p. 85)—and if we introduce two pints of hydrogen and one pint of oxygen into a vessel of the size that would contain the two pints of hydrogen alone at that temperature and pressure, the mixture may remain quiescent for any length of time with no change of temperature or physical quality. If we adopt the molecular theory, we must suppose the molecules of both gases moving with such velocities as to overcome any tendency to cohesion—any attraction—which may exist between either similar or dissimilar molecules. If we suppose (as chemists are for various reasons inclined to believe) that there are as many molecules per pint in all gases at the same temperature and pressure, and if we represent hydrogen molecules by triangles and oxygen by squares (of course without meaning that we know anything about their actual shapes), there will be on an average twice as many triangles as squares in any portion of the vessel; as thus:—

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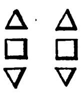
Now, if we introduce into this vessel a piece of platinum, in the state known as 'spongy,' there will follow an explosion, with a great development of heat. And if the vessel be not broken, when all is cooled down again to the original temperature we shall find the platinum just as it was, and *steam* or *water* instead of either hydrogen or oxygen. If the temperature of the original mixture is such as to allow the steam to subsist sensibly in the condition of a perfect gas, it will be at the same pressure as the oxygen or hydrogen was in its separate vessel.

If, instead of a mixture in the exact proportions given above, we had introduced more of one or the other gas, the result would have been the formation of the same quantity of water, with the same development of heat (though less suddenly and explosively), with the residual oxygen or hydrogen, as the case might be, mixed, but not united with the new compound; on cooling it down to ordinary temperature the *water* would sink to the bottom as a liquid, leaving the excess of gas in the space above.

This fact—that a definite number of particles of each substance unite together to form a new permanent chemical substance, leaving the rest unchanged and merely permeating the interspaces of the new compound—the chemical doctrine of *definite proportions*—is one of the main arguments for holding the strictly *atomic* theory; that is, that each simple substance is resolvable into particles indivisible by any chemical action known to us. The result above described seems intelligible as a re-arrangement of the mass such that (taking our proportions as correct) each atom of oxygen unites with two of hydrogen, so as to make a *compound molecule*, coupled together by a closer approximation and stronger force, the whole collection of compound molecules moving still at the *gaseous rates* of velocity, so as practically to act as free bodies impinging against the sides of the vessel independently, and producing the same effect of pressure.



The compound molecule is not conceived as consisting of atoms in contact and at rest, but rather as a system in motion according to its own laws—like a planet with its satellites moving as one element in a larger planetary or solar system.

It remains to show that this theory explains the production of heat, as a rule all but universal, in chemical combination.

The new arrangement of the atoms into *molecules*—using this latter word exclusively for compound collections of so-called atoms, whether these latter be themselves absolutely indivisible or not—is conceived to be *the work of attractive forces* acting between the atoms. Forces doing work produce kinetic energy in proportion to the work done; and the excess of kinetic energy thus introduced into each molecular system will, according to the view already expounded, pass out into the so-called ether—the vehicle of heat, light, &c.—and so become sensible as heat.

The fact that this development of heat obeys the law of conservation tends to make us conceive the forces as permanent and mutual between the elementary atoms, independent of any other condition, such as the velocity and direction of their motion, or the presence of other molecules. But we are here getting to a point where even conjecture, based on any solid ground of reasoning, at present fails us. At least, we may here close our sketch of molecular chemical theory.

VIII. We have still to point out how the connection of friction and inelastic impact with the production of heat is explained. And this is simple, if we combine what has been said (pp. 56, 57) about the vibratory motions left in masses impinging on each other with the theory of heat as the motion communicated from the molecules to the 'ether.'

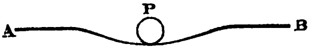
The amount of vibration left in impinging masses supposed perfectly elastic, which would be measured by the loss of kinetic energy in the motion of the masses, as wholes, from place to place, will differ according to the compactness of their composition and the intensity of the forces concerned: the quicker the action, the more it will tell on the masses while still in contact; but some may be expected always to remain in them after separating. Very inelastic bodies will, on this theory, be those in which almost all the kinetic energy of impact passes into molecular vibration.

And if there were no ether, and no contact with air or other bodies, this vibratory motion would go on for ever, like the pendulum or the planetary system, the kinetic energies doing work against the forces, and the forces in turn doing work against the motions. With contact of air, &c. these vibrations would dissipate themselves farther and farther, without altering their essential character as motions of the molecules or atoms of the substances through which they spread.

But these motions, if in excess of what is necessary for equilibrium with the common temperature of the space in which they take place—in other words (on our assumptions), equilibrium with the vibrations of the ether—will be passed on to that medium, and become what we call Heat.

The case of friction is substantially the same.

In a great measure, no doubt, friction is impact of the small irregularities which exist in the smoothest surfaces gliding over one another. But besides this, the molecules of the surfaces are necessarily yielding and rebounding as the masses press on each other and pass on. If the surface A B of any solid be, when no external weight is resting upon it, a horizontal plane, this means that the forces acting between the molecules balance one another with the aid of gravity, which acts on them all, and of the incumbent air. If, then, a weight, P, be placed upon it—however motionless at the moment—this is a new force acting on a system which was before in equilibrium, and its weight must act and descend until, by thus disturbing the arrangement of the molecules, it enables them to exert a resultant force and do work against it; and then there will be an oscillation, up and down, which will spread motion throughout the mass, and finally, as before, pass off into heat.



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